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4. Research Method(s).
5. Results.
6. Discussion of the Results.
7. Acknowledgments, if any.
8. References.
9. Appendix if any.
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Modeling Volatility Spillovers from Developed Stock Markets to Selected African Stock Markets

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Abstract
International stock markets have been characterized by increasing degree of integration. Due to financial linkages between stock markets, volatility could spillover from one to another. The main purpose of this thesis was to model volatility spillovers from developed stock markets to selected African stock markets using MGARCH models. The data on weekly closing return series were obtained from Bloomberg that span from the first week of 1999 to the 46th week of 2017. Among the various MGARCH methodologies often used to analyze volatility spillover, the conditional correlation models such as CCC and DCC have been used in this study. The volatility of stock market returns was captured by Tri-variate DCC (1, 1) model with Student’s –distributional assumption of the disturbances. The results revealed that significant volatility spillovers were detected only from USA to South Africa and from Europe to each of Morocco and South Africa stock market returns. The implication is that the degree of integration and financial linkages of African stock markets with those of the developing world is minimal.

Keywords: DCC; MGARCH; spillover; stock market; volatility

1. Introduction
Stock markets are comparatively new phenomena in Africa. These markets have been functional only in about half of African countries in the last decade. The development of these markets has been considered as an essential component of financial sector development and is expected to play an important role in promoting the growth of African economies (Jefferis, 1995; Piesse and Hearn, 2002). Obviously, how well the African stock markets function plays a crucial role in determining their contributions to economic growth.

International financial markets have been characterized by increasing degrees of integration. However, Sub-Saharan Africa has been lagging behind and its financial markets have been considered fairly independent. Just before the start of the subprime crisis, the Economist characterized Africans as the final frontier of globalization for international investors, suggesting to “Buy Africa” to diversify their risk. Indeed, before the global financial meltdown, African financial markets had experienced a large expansion in a very short period of time. There has been at least one African stock market in the top 10 best-performing markets in the world every year since 1995. In 2004, for example, six African countries
(Ghana, Uganda, Kenya, Egypt, Mauritius and Nigeria) had the world’s 10 best-performing stock markets. In 2005, Egypt, Uganda and Zambia were in the top five (Giovannetti, G., Velucchi, M., 2013).

Several recent studies have examined how new shocks from one international stock market influence the volatility process of other markets. Accordingly, stock market returns are vulnerable to both rising and falling swings. Modeling and forecasting volatility and correlations are now at the heart of financial econometrics as accurate estimates of volatility and correlation are required in derivative pricing, portfolio optimization, risk management and hedging strategies (Sadorsky, 2012). Although the idea that financial markets do influence each other had been well understood (Engle, Ito & Lin, 1990), the growing integration of financial markets has led to renewed interest on stock market interaction and the mechanisms by which stock return movements are transmitted. Some of the recent works include that of Guimarães-Filho & Hong (2016) and Hirayama & Tsutsui (2013) who have analyzed the interaction between Japanese, Korean and Chinese stock markets using volatility and correlation models, and Iryna and Plamen (2013) who have examined the effect and levels of volatility contagion between several stock indices.

In recent years, stock markets in the world have been exhibiting increase interdependence. Due to advances in information technology, financial globalization, etc., stock markets’ volatility could easily spillover from one market to another. However, there is a dearth of work focusing on volatility spillover/contagion between developed stock markets and African stock markets. Thus, this study attempts to investigate volatility spillovers from developed to selected African stock markets using MGARCH models.

The rest of the study is organized as follows. Section 2 reviews the literature with emphasis on the statistical tools relevant to modeling the volatility spillovers from developed to African stock markets. Section 3 discusses the methodology which has been applied in building VAR and MGARCH models and estimating their parameters. The fourth section presents the results of analysis. Section 5 provides a summary of results and conclusions.

2. Literature review

Volatility is a measure of uncertainty about future price or return changes on assets. Regarding the factors which drive volatility, there are two arguments. Some scholars argue that it is exogenously driven by unobservable factors which are correlated with asset returns, while others contend that stock market volatility follows a very strong pattern of business cycle. According to Jones (2002), volatility tends to be higher during recession than during expansion. Economists and policy makers largely believe that
financial globalization has the primary impact of reducing domestic barriers to cross-border financial flows.

Financial globalization is the integration of a country’s local financial system with international financial market. Integration takes place when liberalized economies experience an increase in cross-country capital movement, including an active participation of local borrowers and lenders in international markets and a widespread use of international financial intermediaries. Frenkel (2003) describes financial globalization as a historical process with two dimensions: one is the growing volume of cross border financial transactions; and the other is the sequence of institutional and legal reforms implemented to liberalize and deregulate international capital movements and national financial systems. This move towards free and fast capital flow makes the countries within the global market closely related and dependent upon each other. As a result, a financial crisis in one country can quickly spread to other countries (Dymski, 2005).

The recent financial and credit crises have shifted focus on the interdependence level of financial markets as well as volatility spillovers (Gatfaoui, 2012). International stock markets, under ever expanding globalization, have been experiencing an increasing interdependency or interaction among them as a result of information spillovers among stock markets. The existence of volatility spillovers implies that one large shock increases the volatilities not only in its own asset or market, but also in other assets or markets as well. Volatility changes signal flow and arrival of new information (Ross, 1989). If information comes in clusters, asset returns or prices may exhibit volatility even if the market perfectly and instantaneously adjusts to the news. Thus, study on volatility spillover can help to understand how information is transmitted across stock markets. As a consequence, current literature has increasingly focused on the spillover effect and volatility (Beirne et.al. 2008; Mukherjee and Mishra, 2010; Kumar and Pandey, 2011; among others).

Volatility spillover has stirred an enormous interest amongst researchers and practitioners to develop models that can accurately estimate and forecast volatility. To capture the time-varying feature of conditional correlation between equities and exchange rates, Kroner and Sultan (1993) applied the constant conditional correlation bivariate GARCH (bivariate CCC-GARCH) model to hedge the currency exposure risk. While conditional variance of different assets and currency forward prices change over time, the conditional correlations for currency markets are assumed to be constant in order to get a positive definite variance-covariance matrix as proposed in Bollerslev (1990). This constant-correlation approach has been widely applied because of its computational simplicity. However, financial data of
equities and exchange rates provided strong evidence that the assumption of having a constant correlation was violated for these markets.

Several studies have shown that new family of distributions, the multivariate skew-Student density, combined with a multivariate dynamic conditional correlation (DCC)-GARCH model is useful for modeling financial returns and forecasting the Value-at-Risk (VaR) of portfolios of assets. For instance, Engle (2002) showed that the bivariate version of DCC model provides a very good approximation to a variety of time varying correlation processes. The comparison of DCC with simple multivariate GARCH and several other estimators showed that the DCC is often the most accurate. Thus, in order to investigate volatility spillovers/contagion from developed stock markets to selected African stock markets, this study utilizes the CCC and DCC multivariate GARCH models.

3. Data and methodology

3.1 Source and type of data

This study used stock market data obtained from Bloomberg that span from the first week of 1999 to the 46th week of 2017. The data are on the US dollar denominated weekly closing stock indices, developed economies’ weekly closing indices, and selected African countries weekly closing indices. The weekly stock returns are defined as 100 times the first differences of the logarithm of stock indices. The total number of observations for each stock market was 983.

The four African stock markets selected for analysis were among the 10-top best performing in the world since 1995. These are:

i) Kenya stock indices: The Nairobi Securities Exchange Ltd 20 Share Index is a price weighted index. The members are selected based on a weighted market performance for a 12 month period: Market Capitalization 40%, Shares Traded 30%, Number of deals 20%, and Turnover 10%.

ii) Morocco stock indices: The MASI index is a broad based free float index comprising all shares listed on the Casablanca Stock Exchange.

iii) South African stock indices: The JSE Africa All Shares Index is a market capitalization-weighted index. Companies included in this index make up the top 99% of the total pre free-float market capitalization of all listed companies in the Johannesburg Stock Exchange.

iv) Nigerian stock indices: The Nigerian Stock Exchange all Share Index was formulated in January 1984 with a base value of 100. The All-Share Index tracks the general market movement of all listed equities on the Exchange, including those listed on the Alternative Securities Market (ASeM), regardless of capitalization.
To investigate the presence of volatility spillovers from developed stock markets to selected African stock markets, the study used the following stock indices from the developed world:

i) The Standard & Poor's 500 (S&P 500) Index is a market-capitalization-weighted index of the 500 largest U.S. publicly traded companies by market value. The index is widely regarded as the best single gauge of large-cap U.S. equities.

ii) The S&P Europe 350 index is made up of 350 individual European company stocks drawn from 17 major European markets and represents approximately 70% of the region's market capitalization. Similar to the S&P 500 index in the United States, the S&P Europe 350 index can be used as a benchmark to measure a European stock's performance.

iii) The U.S. dollar index (USDX, DXY, DX) is an index (or measure) of the value of the United States dollar relative to a basket of six different foreign currencies as measured by their exchange rates. Over half of the index's value is represented by the dollar's value measured against the euro. The other five currencies consist of the Japanese yen, the British pound, the Canadian dollar, the Swedish krona, and the Swiss franc.

The vector of endogenous (response) variables ($r_t$) is the weekly closing stock market returns of developed and selected African stock markets. Specifically, $r_t = (r_{t1}, r_{t2}, r_{t3})'$, where $r_{t1}$, $r_{t2}$ and $r_{t3}$ represents the return of stock markets for developed countries (US or Europe), the return of stock markets for selected African countries and US dollar return at time (week) $t$, respectively. The lagged values of weekly closing returns are used as independent variables in our VAR specification without exogenous covariates.

### 3.2 Stationarity and unit-root test

In this study multivariate time series will be used. Multivariate time series involves a vector of time series data that are simultaneously modeled. The first step for an appropriate analysis is to determine whether the time series under consideration are stationary or not. Many economic and financial time series exhibit a trending behavior or non-stationarity in the mean. Due to non-stationarity, regressions with time series data are very likely to result in spurious results. The two tests of stationarity that have become widely popular over the past several years, namely, the Augmented Dickey- Fuller (ADF) test due to Dickey and Fuller (1979), and the Phillip-Perron (PP) test due to Phillips and Perron (1988), are used to test for the existence of unit roots.
3.3 Vector autoregressive (VAR) model

The main purpose of this study is modeling volatility spillovers/contagion from developed stock markets to selected African stock markets. Before estimating the random variance (volatility) model, however, it is necessary to estimate the mean model. Since our response variable is tri-variate, the vector autoregressive (VAR) model is used. VAR models are proposed by Sims (1980) and can be used to capture the dynamic and the interdependency of multivariate time series. In this study a tri-variate VAR (p) model is applied for weekly closing stock market returns. The tri-variate VAR (1) model, for instance, has the form:

$$r_t = C + \Pi r_{t-1} + \epsilon_t$$

where $r_t = (r_{1t}, r_{2t}, r_{3t})'$ is as defined earlier, $C = (c_1, c_2, c_3)'$ is a vector of constants, $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})'$ is a vector of disturbance terms and $\Pi$ is a matrix of regression coefficients. The diagonal elements $\pi_{ii}$ of matrix $\Pi$ quantify the effects of the respective market’s own one-period lagged returns on the contemporaneous returns, while the off-diagonal elements $\pi_{ij}$ $(i \neq j)$ represent the mean spillover effects across markets.

The lag length for the VAR (p) model may be determined using model selection criteria such as Akaike information criterion (AIC), Schwarz–Bayesian information criterion (BIC) and Hannan–Quinn information criterion (HQIC). In VAR models we assume that the error terms in each of the equations are uncorrelated with any of the lagged endogenous (and exogenous) variables. Thus, the usual OLS procedure could be used to estimate the parameters efficiently and consistently. After model selection and parameter estimation, we need to test the adequacy of the model before using it for some specific purposes. These could be checked by examining the residual autocorrelation (auto-covariance) and multivariate normality of the vector of residuals.

3.4 Multivariate GARCH model

To analyze the volatility spillovers/contagion in the three markets (indices), we estimated a multivariate GARCH (MGARCH) model which is an extension of univariate GARCH models. It helps us to capture the dynamic relationship between developed stock markets and selected African stock markets. The specification of an MGARCH model should be flexible enough to be able to represent the dynamics of...
the conditional variances and covariances on one hand. On the other hand, it should be parsimonious enough to allow for relatively easy estimation and interpretation of the model parameters. Another feature that needs to be taken into account in the specification is imposing positive definiteness of the conditional covariance matrices. One possibility is to derive conditions under which the conditional covariance matrices implied by the model are positive definite (but this is often infeasible in practice). An alternative is to formulate the model in a way that positive definiteness is implied by the model structure.

In its most general specification, the MGARCH model takes the following form:

\[
\mathbf{r}_t = \mathbf{C} + \Pi \mathbf{r}_{t-1} + \mathbf{\varepsilon}_t
\]

\[
\mathbf{\varepsilon}_t = \mathbf{H}_t^{1/2} \mathbf{Z}_t
\]

(2)

where \( \{\mathbf{Z}_t\} \) is a sequence of iid random vectors with \( \mathbb{E}(\mathbf{Z}_t) = \mathbf{0} \) and \( \mathbb{E}(\mathbf{Z}_t \mathbf{Z}_t') = \mathbf{I}_n \), \( \mathbb{E}(\mathbf{\varepsilon}_t | \mathbf{\psi}_{t-1}) = \mathbf{0} \) and \( \mathbb{E}(\mathbf{\varepsilon}_t \mathbf{\varepsilon}_t' | \mathbf{\psi}_{t-1}) = \mathbf{H}_t \). Here \( \mathbf{\psi}_{t-1} \) denotes the information set up to and including time \( (t-1) \). In this study, the residual vector \( \mathbf{\varepsilon}_t \) is tri-variate and its corresponding conditional variance-covariance matrix is given by:

\[
\mathbf{H}_t = \begin{bmatrix}
    h_{11,t} & h_{12,t} & h_{13,t} \\
    h_{21,t} & h_{22,t} & h_{23,t} \\
    h_{31,t} & h_{32,t} & h_{33,t}
\end{bmatrix}
\]

(3)

Multivariate volatility models provide a parametric structure for the dynamic evolution of \( \mathbf{H}_t \). The assumptions regarding \( \mathbf{H}_t \) are:

i) The diagonal elements of \( \mathbf{H}_t \) are strictly positive;

ii) \( \mathbf{H}_t \) is positive definite;

iii) Stationarity: \( \mathbb{E}(\mathbf{H}_t) \) exists, is finite and constant with respect to \( t \).

There are various parametric formulations of MGARCH models. These include the conditional covariance matrix model, the factor models, and models for conditional variances and correlations, among others. The Constant Conditional Correlations (CCC) and the Dynamic Conditional Correlations (DCC) models, which are members of the latter class of models, are utilized in this study.

3.4.1 Constant conditional correlations (CCC) models

The CCC model was introduced by Bollerslev (1990) to model primarily the conditional covariance matrix indirectly by estimating the conditional correlation matrix. In CCC models, \( \mathbf{H}_t \) is decomposed
into a matrix of time invariant conditional correlations $R$ and a diagonal matrix of conditional variances $D_t$:

$$H_t = D_t^{1/2} R D_t^{1/2} \quad \text{.................................................. (4)}$$

In the tri-variate case, the above decomposition is given by:

$$
\begin{bmatrix}
    h_{11,t} & h_{12,t} & h_{13,t} \\
    h_{21,t} & h_{22,t} & h_{23,t} \\
    h_{31,t} & h_{32,t} & h_{33,t}
\end{bmatrix}
= 
\begin{bmatrix}
    \sigma_{1t} & 0 & 0 \\
    0 & \sigma_{2t} & 0 \\
    0 & 0 & \sigma_{3t}
\end{bmatrix}
\begin{bmatrix}
    1 & \rho_{12} & \rho_{13} \\
    \rho_{12} & 1 & \rho_{23} \\
    \rho_{13} & \rho_{23} & 1
\end{bmatrix}
\begin{bmatrix}
    \sigma_{1t} & 0 & 0 \\
    0 & \sigma_{2t} & 0 \\
    0 & 0 & \sigma_{3t}
\end{bmatrix}
= 
\begin{bmatrix}
    \sigma_{1t} & 0 & 0 \\
    0 & \sigma_{2t} & 0 \\
    0 & 0 & \sigma_{3t}
\end{bmatrix}
$$

implies that $h_{ij,t} = \rho_{ij} \sqrt{h_{ii,t} h_{jj,t}}$, $i \neq j$, $i, j = 1, 2, 3$. Here the diagonal elements $h_{ii,t} = \sigma_{it}^2$ follow univariate GARCH processes:

$$h_{ii,t} = \alpha_{i0} + \sum_{q=1}^{Q} \alpha_{iq} \varepsilon_{i,t-q}^2 + \sum_{p=1}^{P} \beta_{ip} h_{ii,t-p} \quad , i = 1, 2, 3 \quad \text{.......................... (5)}$$

3.4.2 Dynamic conditional correlations (DCC) models

Although the CCC–GARCH model in many respects has an attractive parameterization, empirical studies have suggested that the assumption of constant conditional correlations may be too restrictive. The model may therefore be generalized by retaining the previous decomposition but making the conditional correlation matrix time-varying. In the DCC model proposed by Engle (2002), the diagonal elements of $H_t$ are modeled as univariate GARCH processes, while the off-diagonal elements are modeled as nonlinear functions of the diagonal terms:

$$h_{ij,t} = \rho_{ij} \sqrt{h_{ii,t} h_{jj,t}} \quad , i \neq j \quad i, j = 1, 2, \ldots, n \quad \text{.......................... (6)}$$

where $\rho_{ij,t}$ follows a dynamic process rather than being constrained to be constant as in the CCC specification. The DCC model may be written as:

$$r_t = C + \prod r_{t-1} + \varepsilon_t$$

$$\varepsilon_t = H_t^{1/2} z_t$$

$$H_t = D_t^{1/2} R_t D_t^{1/2}$$

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t^{-1/2} \text{diag}(Q_t)^{-1/2}$$

$$Q_t = \lambda_1 \tilde{e}_{t-1} \tilde{e}_{t-1} + \lambda_2 Q_{t-1} \quad \text{.......................... (7)}$$
where $D_t$ is a diagonal matrix of conditional variances, $R_t$ is a matrix of conditional quasi-correlations, $\tilde{\varepsilon}_t$ is a vector of standardized residuals ($D_t^{-1/2}\varepsilon_t$), and $R$ is the unconditional covariance matrix of $\tilde{\varepsilon}_t$. The two additional parameters, $\lambda_1$ and $\lambda_2$, are adjustment parameters that govern the evolution of the conditional quasi-correlations. To ensure that $Q_t$ is positive definite, we impose the conditions that both $\lambda_1$ and $\lambda_2$ are non-negative satisfying $\lambda_1 + \lambda_2 < 1$. In addition, $Q_0$, the starting value of $Q_t$, has to be positive definite to guarantee that $H_t$ is positive definite. The DCC model reduces to the CCC model when the adjustment parameters that govern the dynamic correlation process are jointly equal to zero (that is, $\lambda_1 = \lambda_2 = 0$). For further details, one may refer to Caporin and McAleer (2009, 2012).

In the tri-variate case, the decomposition of $H_t$ is given by:

$$
\begin{bmatrix}
h_{11,t} & h_{12,t} & h_{13,t} \\
h_{21,t} & h_{22,t} & h_{23,t} \\
h_{31,t} & h_{32,t} & h_{33,t}
\end{bmatrix} =
\begin{bmatrix}
\sigma_{1t} & 0 & 0 \\
0 & \sigma_{2t} & 0 \\
0 & 0 & \sigma_{3t}
\end{bmatrix}
\begin{bmatrix}
1 & \rho_{12,t} & \rho_{13,t} \\
\rho_{12,t} & 1 & \rho_{23,t} \\
\rho_{13,t} & \rho_{23,t} & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_{1t} & 0 & 0 \\
0 & \sigma_{2t} & 0 \\
0 & 0 & \sigma_{3t}
\end{bmatrix}
$$

\[ \text{............. (9)} \]

3.5 Parameter estimation

It is common to estimate MGARCH models by maximizing a multivariate Gaussian likelihood function. This estimation procedure, called the Gaussian Quasi-Maximum Likelihood (QML) estimation, can produce consistent parameter estimates even when the true distribution is not Gaussian. This is an important result since it is common to find that, after correcting the returns for the dynamics in the conditional covariance matrix, the marginal distribution of the standardized return series may still be heavy tailed. Furthermore, when dealing with models with conditional heteroskedasticity, the estimates are known to be asymptotically normal (Bollerslev & Wooldridge, 1992). The other distributional assumptions of the standardized residuals often considered are the multivariate Student’s t- and a multivariate skew Student’s t-distribution.

4. Results and discussion

4.1 Descriptive statistics and time series plots

Table 1 presents some descriptive statistics on the returns of US dollar denominated weekly closing stock indices, developed economies’ weekly closing indices, and selected African weekly closing indices. The average weekly returns (except US dollar) were positive – an indication of the increasing trend in the stock markets’ return. We can also observe excess kurtosis for all series under consideration (the kurtosis
statistics are far greater than three). The Jarque-Bera tests reject the null hypothesis that the returns are normally distributed for all stock market returns at 5% level of significance.

Table 1: Descriptive statistics

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<th>Statistics</th>
<th>USA</th>
<th>Europe</th>
<th>US Dollar</th>
<th>Morocco</th>
<th>Kenya</th>
<th>Nigeria</th>
<th>South Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0878</td>
<td>0.0248</td>
<td>-0.0007</td>
<td>0.0898</td>
<td>0.0236</td>
<td>0.0558</td>
<td>0.1618</td>
</tr>
<tr>
<td>St. dev.</td>
<td>2.4436</td>
<td>2.923</td>
<td>1.1233</td>
<td>2.2939</td>
<td>2.4266</td>
<td>3.6898</td>
<td>3.7414</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.3292</td>
<td>-0.7131</td>
<td>0.2097</td>
<td>-0.5515</td>
<td>0.1175</td>
<td>-0.7792</td>
<td>-0.4754</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.1574</td>
<td>7.0798</td>
<td>3.9425</td>
<td>8.074</td>
<td>7.6026</td>
<td>8.4519</td>
<td>6.3813</td>
</tr>
</tbody>
</table>

Time series plots for weekly stock market returns are shown in Figure A (Annex). The plots exhibit volatility clustering (persistence) behaviour: large absolute returns tend to be followed by large absolute returns and vice versa. Moreover, the plots show that the volatilities of developed and selected African returns tend to move in the same fashion. Around 2008, for instance, large fluctuations in the stock returns of the developed world were accompanied by considerable volatility in the returns of selected African stock markets. This is probably an indication of volatility spillovers from developed stock returns to African stock market returns.

4.2 Tests of stationarity

Table 2 shows the results of Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests for developed and selected Africa stock market returns. From the results we can observe that the null hypothesis of a unit root is rejected for all return series under consideration, that is, all return series are stationary, and we can proceed to estimation of mean (VAR) models.

Table 2: Unit-root test results for stock market return series

<table>
<thead>
<tr>
<th>Stock market</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test statistic</td>
<td>p-value</td>
</tr>
<tr>
<td>US Dollar</td>
<td>-30.384</td>
<td>0.000</td>
</tr>
<tr>
<td>USA</td>
<td>-35.746</td>
<td>0.000</td>
</tr>
<tr>
<td>Morocco</td>
<td>-28.792</td>
<td>0.000</td>
</tr>
<tr>
<td>Kenya</td>
<td>-26.621</td>
<td>0.000</td>
</tr>
<tr>
<td>Nigeria</td>
<td>-31.381</td>
<td>0.000</td>
</tr>
<tr>
<td>South Africa</td>
<td>-32.876</td>
<td>0.000</td>
</tr>
<tr>
<td>European</td>
<td>-34.253</td>
<td>0.000</td>
</tr>
</tbody>
</table>
4.3 Model selection

The first stage in specifying the mean (VAR) model is the selection of the appropriate order based on information criteria. Table 3 presents the results of candidate tri-variate VAR(p) models together with serial correlation and ARCH tests of residuals where the developed stock market returns are represented by that of USA. The tri-variate VAR (1) model has the minimum AIC and BIC for Morocco, Kenya and South Africa returns, while VAR (3) model is the best-fit model for Nigeria returns. The Breusch–Godfrey serial correlation LM test results indicate that there is no serial correlation in the residuals of the mean equation for all return series under consideration. Moreover, the null hypothesis of no ARCH effects in the residuals from the mean equations is rejected at 1% level of significance for Morocco, Kenya and South Africa returns. This implies that the conditional covariance of the weekly return series of US stock market and African stock markets (except Nigeria) is not time invariant, and hence, should be modeled by MGARCH family (CCC and DCC) models.

Table 3: VAR order selection and tests of model adequacy & ARCH effects (US returns)

<table>
<thead>
<tr>
<th>VAR(p)</th>
<th>Developed stock market</th>
<th>African stock market</th>
<th>US Dollar</th>
<th>AIC</th>
<th>BIC</th>
<th>Serial correlation test</th>
<th>ARCH test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>USA</td>
<td>Morocco* USD</td>
<td>11.8878</td>
<td>11.9478</td>
<td>0.5318</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya* USD</td>
<td>12.2256</td>
<td>12.2858</td>
<td>0.6862</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria USD</td>
<td>13.1081</td>
<td>13.1686</td>
<td>0.0007</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa* USD</td>
<td>12.4200</td>
<td>12.4802</td>
<td>0.7837</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>USA</td>
<td>Morocco USD</td>
<td>11.9003</td>
<td>12.0054</td>
<td>0.6281</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya USD</td>
<td>12.2400</td>
<td>12.3456</td>
<td>0.2384</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria USD</td>
<td>13.0951</td>
<td>13.2013</td>
<td>0.0082</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa USD</td>
<td>12.4330</td>
<td>12.5387</td>
<td>0.7358</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>USA</td>
<td>Morocco USD</td>
<td>11.9044</td>
<td>12.0546</td>
<td>0.9698</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya USD</td>
<td>12.2245</td>
<td>12.3758</td>
<td>0.6364</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria* USD</td>
<td>13.1018</td>
<td>13.2541</td>
<td>0.0826</td>
<td>0.1196</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa USD</td>
<td>12.4428</td>
<td>12.5943</td>
<td>0.1286</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results in Table 4, where the developed stock market returns are represented by that of Europe, are almost similar to those reported in Table 3.

4.4 Model Comparison

The choice of the conditional correlation model in this study was made based on the adjustment parameters. The DCC MGARCH model reduces to the CCC MGARCH model when $\lambda_1 = \lambda_2 = 0$. The Wald test rejects the null hypothesis that $\lambda_1 = \lambda_2 = 0$ at the conventional levels of significance for all developed-African stock returns combinations. These results indicate that the assumption of time-
invariant conditional correlations maintained in the CCC MGARCH model is too restrictive for these data.

Table 4: VAR order selection and tests of model adequacy& ARCH effects (Europe returns)

<table>
<thead>
<tr>
<th>VAR(p)</th>
<th>Developed stock market</th>
<th>African stock market</th>
<th>USD</th>
<th>AIC</th>
<th>BIC</th>
<th>Serial correlation test</th>
<th>ARCH test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Europe</td>
<td>Morocco*</td>
<td>USD</td>
<td>12.0846</td>
<td>12.1445</td>
<td>0.3288</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya*</td>
<td></td>
<td>12.4177</td>
<td>12.4780</td>
<td>0.3322</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria</td>
<td></td>
<td>13.2926</td>
<td>13.3531</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa*</td>
<td></td>
<td>12.3860</td>
<td>12.4463</td>
<td>0.1544</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>Europe</td>
<td>Morocco</td>
<td>USD</td>
<td>12.9605</td>
<td>12.2012</td>
<td>0.6281</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya</td>
<td></td>
<td>12.4314</td>
<td>12.5373</td>
<td>0.6457</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria</td>
<td></td>
<td>13.2773</td>
<td>13.3835</td>
<td>0.0125</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa</td>
<td></td>
<td>12.3925</td>
<td>12.4983</td>
<td>0.7089</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Europe</td>
<td>Morocco</td>
<td>USD</td>
<td>12.1059</td>
<td>12.2564</td>
<td>0.9954</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kenya</td>
<td></td>
<td>12.4265</td>
<td>12.5781</td>
<td>0.3899</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nigeria*</td>
<td></td>
<td>13.2851</td>
<td>13.4374</td>
<td>0.0556</td>
<td>0.1196</td>
</tr>
<tr>
<td></td>
<td></td>
<td>South Africa</td>
<td></td>
<td>12.4023</td>
<td>12.5541</td>
<td>0.2620</td>
<td></td>
</tr>
</tbody>
</table>

* selected mean VAR(p) model based on AIC and BIC

To capture the volatility spillover in the stock markets using the DCC-GARCH models, the multivariate normal and Student’s t- distributional assumptions of the standardized residuals were considered. The log-likelihood statistics in Table 5 show that the tri-variate DCC-GARCH (1,1) models with Student’s distribution (with six degree of freedom) are the preferred models for all vectors of stock market returns under consideration.

Table 5: Log-likelihood statistics under normal and t-distributional assumptions of standardized residuals

<table>
<thead>
<tr>
<th>Developed stock market</th>
<th>African stock market</th>
<th>US Dollar</th>
<th>t-dist. (v=6)</th>
<th>normal distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>Morocco</td>
<td>-5512.425</td>
<td>-5571.155</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>Kenya</td>
<td>-5640.526</td>
<td>-5710.858</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>South Africa</td>
<td>-5795.788</td>
<td>-5829.871</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>Morocco</td>
<td>-5614.601</td>
<td>-5669.039</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>Kenya</td>
<td>-5741.65</td>
<td>-5807.344</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>South Africa</td>
<td>-5789.077</td>
<td>-5825.141</td>
<td></td>
</tr>
</tbody>
</table>
4.5 Discussion of fitted DCC-GARCH models

The fitted tri-variate DCC-GARCH (1, 1) models for stock returns of the developed world (USA and Europe), selected African stock returns and US Dollar markets are shown in Tables 6 and 7. Checking the adequacy of the fitted MGARCH models is essential for identifying possible model misspecifications and drawing valid inferences. The Ljung–Box portmanteau test revealed that there was no serial correlation in the vectors of standardized residuals and squared standardized residuals after fitting the tri-variate DCC model with Student’s t distributional assumption. The standardized errors were also found to be approximately normally distributed.

4.5.1 GARCH-DCC (1,1) model for USA and selected African returns

Table 6 reports the results of the estimated tri-variate DCC-GARCH (1, 1) model for USA stock returns, selected African stock returns and US Dollar markets. We can observe that there are significant GARCH effects, and that the estimated GARCH parameters are considerably larger than the corresponding ARCH coefficients. This indicates that the conditional variances of market returns are more influenced by their own lagged values rather than by “fresh news” which are reflected by the lagged innovations. Moreover, the sums of the estimated ARCH and GARCH parameters are close to one – an indication of high persistence in conditional covariance. The adjustment parameters (λ₁ and λ₂) that govern the evolution of the conditional quasi-correlations are significant for all US-African stock market combinations. By comparing the estimated values of these two parameters we can conclude that the evolution of the conditional covariances depends more on their past values than on lagged residuals’ innovations.

The DCC results suggest the existence of a dynamic and time varying correlation between US and South African stock returns. The positive and statistically significant conditional correlation between the two stock returns is an indication that an increase in the volatility of US stock returns leads to an increase in the volatility of South African stock returns and vice versa, that is, there are volatility spillovers from USA to South Africa. The conditional correlations between US stock returns and each of Morocco and Kenya stock returns are statistically insignificant. The implication is that there are no volatility spillovers from USA to these two stock returns.

4.5.2 GARCH-DCC (1,1) model for Europe and selected African returns

The results of the estimated tri-variate DCC-GARCH (1, 1) model for European stock returns, selected African stock returns and US Dollar markets are shown in Table 7. The GARCH (1,1) parameters are statistically significant for all European-African stock market volatility models. Moreover, the
significance of the adjustment parameters revealed the presence of a time-varying variance-covariance process.

Table 6: Estimation results of GARCH-DCC (1,1) model for USA and selected African returns

<table>
<thead>
<tr>
<th>Stock market</th>
<th>Selected Africa</th>
<th>Us dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Z-value</td>
</tr>
<tr>
<td></td>
<td>USA</td>
<td>Morocco</td>
</tr>
<tr>
<td>C</td>
<td>0.13781</td>
<td>0.22680</td>
</tr>
<tr>
<td>Arch1</td>
<td>0.12778</td>
<td>0.10014</td>
</tr>
<tr>
<td>Garch1</td>
<td>0.85693</td>
<td>0.85602</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.03325</td>
<td>0.52</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.02388</td>
<td>3.39</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.94536</td>
<td>54.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stock market</th>
<th>Selected Africa</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Z-value</td>
</tr>
<tr>
<td></td>
<td>USA</td>
<td>Kenya</td>
</tr>
<tr>
<td>C</td>
<td>0.14128</td>
<td>0.02565</td>
</tr>
<tr>
<td>Arch1</td>
<td>0.12014</td>
<td>0.04892</td>
</tr>
<tr>
<td>Garch1</td>
<td>0.82395</td>
<td>0.93816</td>
</tr>
<tr>
<td>Correlation</td>
<td>-0.04610</td>
<td>-0.64</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.02993</td>
<td>4.00</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.94090</td>
<td>53.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stock market</th>
<th>Selected Africa</th>
<th>South Africa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Z-value</td>
</tr>
<tr>
<td></td>
<td>USA</td>
<td>South Africa</td>
</tr>
<tr>
<td>C</td>
<td>0.09096</td>
<td>0.37195</td>
</tr>
<tr>
<td>Arch1</td>
<td>0.09694</td>
<td>0.05802</td>
</tr>
<tr>
<td>Garch1</td>
<td>0.87434</td>
<td>0.91821</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.54604</td>
<td>8.21</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.03578</td>
<td>5.98</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.94185</td>
<td>96.23</td>
</tr>
</tbody>
</table>

From Table 7 we can also see that the estimated conditional correlations between European and each of Morocco and South Africa stock returns are positive and statistically significant. This is an indication of volatility spillover from developed stock markets to these two African stock markets. The volatility of European stock market return and those of Morocco and South Africa tend to move in the same direction.
Table 7: Estimation results of GARCH-DCC (1,1) model for Europe and selected African returns

<table>
<thead>
<tr>
<th>Stock market</th>
<th>Selected Africa</th>
<th>Us dollar</th>
<th>Coefficient</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>European USD</td>
<td></td>
<td></td>
<td>C</td>
<td>0.58908</td>
<td>0.27113</td>
</tr>
<tr>
<td></td>
<td>Morocco</td>
<td></td>
<td>Arch1</td>
<td>0.17719</td>
<td>0.11067</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Garch1</td>
<td>0.76589</td>
<td>0.83933</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Correlation</td>
<td>0.22849</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_1$</td>
<td>0.03103</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_2$</td>
<td>0.94560</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>European USD</th>
<th>Kenya</th>
<th></th>
<th>Coefficient</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>C</td>
<td>0.56608</td>
<td>0.69379</td>
</tr>
<tr>
<td></td>
<td>Kenya</td>
<td></td>
<td>Arch1</td>
<td>0.17126</td>
<td>0.22863</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Garch1</td>
<td>0.77350</td>
<td>0.65910</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Correlation</td>
<td>-0.00386</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_1$</td>
<td>0.03356</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_2$</td>
<td>0.94493</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>European USD</th>
<th>South Africa</th>
<th></th>
<th>Coefficient</th>
<th>Z-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>C</td>
<td>0.34037</td>
<td>0.66257</td>
</tr>
<tr>
<td></td>
<td>South Africa</td>
<td></td>
<td>Arch1</td>
<td>0.11967</td>
<td>0.08034</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Garch1</td>
<td>0.84401</td>
<td>0.87796</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Correlation</td>
<td>0.74131</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_1$</td>
<td>0.04404</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_2$</td>
<td>0.93389</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusion
Volatility is a measure of uncertainty about future asset price or return changes. It is a significant input in risk management, strategic financial planning and policy modeling. Financial contagion can create financial volatility and can seriously damage the economy and financial systems of countries.

The main purpose of this study was modeling volatility spillovers from developed stock markets to selected African stock markets using MGARCH models. First, tri-variate VAR models were fitted for...
weekly returns of developed & selected African stock markets and US dollar returns. The best-fit trivariate VAR (3) model for Nigeria stock returns exhibited no ARCH effects, and hence, no further analysis was undertaken. Secondly, volatility spillover from developed stock markets to selected African stock markets was captured by conditional correlation models. Among the candidate conditional correlation models, the Tri-variate GARCH-DCC (1, 1) model with Student’s-t distributional assumption for the residuals was found to be the best-fit for all developed-African stock returns combinations.

The results from the fitted tri-variate GARCH-DCC (1, 1) models revealed that there are statistically significant volatility spillovers from US to South Africa stock markets and from Europe to each of South Africa and Morocco stock markets. The conditional correlations for the remaining developed-African stock market pairs were found to be insignificant. The implication is that the degree of integration and financial linkage of African stock markets with those of the developing world is minimal.

References

Annex

Figure A: Time series plots of weekly return series
Factors Associated with Time to Death of HIV Patients Treated in Alamata Hospital, North Ethiopia: An Application of Bayesian Survival Models

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Abstract

Background: HIV/AIDS remains a major public health problem in Ethiopia and is heavily affecting people of productive and reproductive age. The main objective of this study was to identify potential factors associated with time-to-death of HIV patients on ART follow-up in Alamata General Hospital, North Ethiopia.

Methods: A total of 320 HIV patients were included in the study. Various parametric survival models (Exponential, Weibull, Log-normal, Log-logistic, Gompertz and Generalized gamma distributions) were considered to identify and analyze potential risk factors associated with time-to-death for HIV infected patients under HAART.

Result: During the follow-up period, 89 (27.81%) deaths were registered. The median survival time of HIV patients was 79 months. The Kaplan-Meier survivor estimates for the two sex groups revealed that female patients have longer survival time in comparison with their male counterparts. The mean CD4 cell counts for HIV/AIDS patients have increased from the baseline figure of 126.01 to 305.74 after three years of ART follow-up. Log-normal distribution was found to be more appropriate among the various parametric survival models considered in the study through graphical and non-graphical procedures. Age, functional status, TB screen, past opportunistic infection, baseline CD4 cell counts, WHO clinical stage, sex, marital status, employment status, type of occupation, and baseline weight were found to be statistically significant correlates of time-to-death for HIV infected patients under ART follow-up.

Conclusions: Stakeholders should pay attention to the identified potential predictors so as to reduce mortality of HIV/AIDS patients.

Keywords: HIV; ART; Parametric Survival Models; AFT; Bayesian analysis
1. Introduction

Worldwide, HIV/AIDS has created an enormous challenge on the survival of mankind. Since the recognition of the epidemic, more than 70 million people have been infected with the HIV virus and about 35 million people have died because of HIV. Globally, 36.7 million people were living with HIV at the end of 2016. In 2017, 940,000 people have died due to HIV-related illnesses worldwide. An estimated 0.8% of adults aged 15 - 49 years worldwide are living with HIV, although the burden of the epidemic continues to vary considerably between countries and regions. Sub-Saharan Africa remains the most severely affected region where nearly one in every 25 adults (4.2%) are living with HIV and accounting for nearly two-thirds of the people living with HIV worldwide (WHO, 2017).

Even if the HIV prevalence rate in Ethiopia is lower than other Sub-Saharan countries, the epidemic remains a major health problem of the nation and heavily affecting people of productive and reproductive age. Based on a single point estimate, there were nearly 1.2 million people living with HIV/AIDS in Ethiopia. The adult prevalence rate was estimated to be 2.4% and the incidence rate was 0.29%. The prevalence and incidence rates significantly vary between geographical areas and gender. The urban prevalence rate is estimated at 7.7%, while the rural prevalence rate is 0.9%. The prevalence rates were 1.7% for males and 2.6% for females (WHO, 2016).

A new report by UNAIDS shows that 19 million of the 35 million people living with HIV globally do not know their HIV-positive status (UNAIDS, 2014). In the HAART era, however, morbidity and mortality of people living with HIV/AIDS has been reduced significantly in both industrialized and less developed regions. HAART has significantly increased the probability of survival and reduced the risk of death for HIV/TB co-infected patients (Nahid et al., 2007; Fairall et al., 2008).

Globally, about 54% of adults and 43% of children living with HIV are currently receiving lifelong antiretroviral therapy (ART). Global ART coverage for pregnant and breastfeeding women living with HIV is high at 76% (WHO, 2017). Anti-retroviral treatment was formally started in Ethiopia in 2003 with cost sharing arrangement. In 2005, the country initiated free ART with the support of Global Fund for Tuberculosis, AIDS and Malaria (GFTAM), and the U.S. Presidency Emergency Plan for AIDS Relief (PEPFAR). In the study area, free ART was started in August 2006 just a year after its implementation in other parts of the nation. A total of 5473 HIV patients were under ART follow-up during the study period. However, the effectiveness of ART initiatives in the study area, and especially how much the intervention was supportive to prolong the survival of HIV patients, was not yet fully investigated. Thus, this study has focused on survival analysis of HIV patients.
Survival analysis involves the modeling of time to event data. In the context of this study, death or failure was considered as an “event”. Although there is a fairly rich literature on semi-parametric survival models (Cox Proportional Hazards) and parametric survival models (Exponential, Weibull, Log-normal, Log-logistic, Generalized Gamma distributions, etc.), more recently Bayesian analysis of survival data has received high attention (Ibrahim et al., 2001).

Thus, the main purpose of the current study was to model the survival of HIV infected patients who are under ART follow up using Bayesian approaches. The remaining part of the paper is organized as follows. Section 2 describes the materials and methods. The basic findings of the study are presented and discussed in Section 3. Finally, concluding remarks are provided in Section 4.

2. Methodology

2.1 Data description and method of data collection

A retrospective cohort study was conducted based on data from HIV patients' intake forms and their follow-up cards on HAART. The patients' forms have been structured by FMoH for uniform use across the country. The patients included in this study were all HIV infected patients who were older than 15 years, started the HAART treatment in 2008/09, and followed up till the end of 2014/15 in Alamata General Hospital. Each patient had a chart/record with a distinctive identification number (known as the ART unique identification number). The number of HIV infected patients who started ART in 2008/9 was 515 patients. This study covers a sample of 320 HIV infected patients who attended ART medication program in Alamata General Hospital.

2.2 Sampling technique and sample size determination

In this study, the sampling frame was obtained from the follow-up list at Alamata General Hospital and simple random sampling procedure was applied to select the sample for analysis. The hospital’s appointment record logbook was used to select the patients.

For survival data, power is entirely driven by the number of events. In this study, we considered $\beta = 0.2$ (power = 0.8), $\alpha = 0.05$ and equal allocation ($\pi_1 = \pi_2 = 0.5$) for the gender of HIV patients. The required number of events is computed as:

$$\text{Events} = \frac{(Z_{\alpha/2} + Z_{\beta})^2}{\pi_1 \pi_2 [\ln(\text{HR})]^2}$$
where $Z_{\alpha/2}$ and $Z_{\beta}$ are standard normal percentiles. Plugging in the respective figures we have:

$$\text{Events} = \frac{(1.96 + 0.842)^2}{(0.5)(0.5)[\ln(0.5)]} = 65.365 \approx 66$$

Next, we need to consider the probability of an event during the study. The probability of an event could be computed as:

$$P(\text{Event}) = 1 - [\pi_1 S_1(t) + \pi_2 S_2(t)]$$

We can get the values of $S_1(t)$ and $S_2(t)$ by assuming exponential survival times, that is, $S(t) = \exp(-\lambda t)$. During the follow-up years ($T = 8$), equal allocation estimates of the incidence rate (IR) for one group was 0.3122 events/person-eight-years (or 0.039 events/person-one-year). Thus,

$$S_1(T) = \exp(-0.3122) = 0.732$$
$$S_2(T) = \exp(-0.3122 \times 0.5) = 0.855$$

$$P(\text{Event}) = 1 - [0.5 \times 0.732 + 0.5 \times 0.855] = 0.2063$$

Once we have this probability, the total number of people can then essentially be calculated as:

$$n = \frac{\text{Events}}{P(\text{Event})} = \frac{66}{0.2063} = 319.92 \approx 320$$

Hence, the required sample size for the study was 320.

### 2.3 Study variables

a) **Response variable**

The dependent variable was the survival time of HIV infected patients from the date of initiation of HAART up to the end of the study (in months). Death was considered as the event of the study and the response time is the time when the patient died. HIV infected patients under HAART who were alive up to the end of the study, lost, dropped out or died due to other causes were considered as censored.

b) **Independent variables**

The independent variables considered in this study were classified as demographic, socio-economic and other variables. Specifically, the independent variables included in the study are age, gender, baseline
weight, CD4 cell counts, functional status, regimen, TB screen, past opportunistic infection, WHO clinical stage, level of education, marital status, religion, residence and occupation.

2.4 Parametric Survival Models
Parametric survival models assume that the survival times follow a specified probability distribution and that the parameters of that distribution depend on covariates. Among the popular parametric survival regression models, this study has considered Weibull, Exponential, Gompertz, Log-normal, Log-logistic and Generalized gamma distributions.

2.4.1 Weibull and Exponential Models
The Weibull and exponential models are parameterized as both Proportional hazard (PH) and Accelerated Failure Time (AFT) models. The Weibull distribution is suitable for modeling data with monotone hazard rates that either increase or decrease exponentially with time, whereas the exponential distribution is suitable for modeling data with constant hazard. For PH model, \( h(t) = \lambda \) for exponential regression and \( h(t) = (p/\lambda)(t/\lambda)^{p-1} \) for Weibull regression, where \( \lambda \) is the scale parameter and \( p \) is shape parameter. The log of the Weibull hazard is a linear function of log time with slope \( (p-1) \). Thus, the hazard is rising if \( p > 1 \), constant if \( p = 1 \), and declining if \( p < 1 \) (Lee & Wang, 2003; Collett, 2014).

2.4.2 Gompertz Models
The Gompertz regression is parameterized only as a PH model. Gompertz distribution is a two-parameter function with the survivor function:

\[
S(t) = \exp[-\lambda \gamma^{-1}(\exp(\gamma t) - 1)]
\]

The model is implemented by the parameterization \( \lambda_j = \exp(X_j\beta) \), implying that \( h_0(t) = \exp(\gamma t) \), where \( \gamma \) is an ancillary parameter to be estimated from the data. When \( \gamma \) is positive, the hazard function increases with time; when \( \gamma \) is negative, the hazard function decreases with time; and when \( \gamma \) is zero, the hazard function is equal to \( \lambda \) for all \( t \), so the model reduces to an exponential function (Collett, 2014).

2.4.3 Log-normal and log-logistic models
The log-normal and log-logistic models are implemented only in the AFT form. These two distributions are similar and tend to produce comparable results. For the log-normal distribution, the natural logarithm of time follows a normal distribution; for the log logistic distribution, the natural logarithm of time follows a logistic distribution. The log-normal survivor function is:
\[ S(t) = 1 - \Phi \left( \frac{\log(t) - \mu}{\sigma} \right) \]

where \( \Phi(z) \) is the standard normal cumulative distribution function. The log-normal regression is implemented by setting \( \mu_j = X_j \beta \) and treating the standard deviation \( (\sigma) \) as an ancillary parameter to be estimated from the data. The log-logistic regression is obtained if \( z \) has a logistic density. The log-logistic survivor function is:

\[ S(t) = \left(1 + (\lambda t)^{1/\gamma} \right)^{-1} \]

This model is implemented by parameterizing \( \lambda_j = \exp(X_j \beta) \) and treating the scale parameter \( \gamma \) as an ancillary parameter which is estimated from the data (Lee & Wang, 2003; Collett, 2014).

\subsection{2.4.4 Generalized gamma model}

The generalized gamma model is implemented only in the AFT form. The probability density function of the generalized gamma distribution is an extension of the gamma density that includes an additional parameter \( \theta > 0 \), and is defined by:

\[ f(t) = \frac{\theta \lambda^{\theta} t^{\gamma-1} \exp[-(\lambda t)^\theta]}{\Gamma(\gamma)} \quad 0 \leq t < \infty \]

The survival function for this distribution is defined in terms of the incomplete gamma function and is given by:

\[ S(t) = 1 - \Gamma_{(\lambda t)^\theta}(\gamma) \]

where

\[ \Gamma_{(\lambda t)^\theta}(\gamma) = \frac{1}{\Gamma(\gamma)} \int_0^{(\lambda t)^\theta} u^{\gamma-1} e^{-u} du \]

Here \( \theta \) is the shape parameter of the distribution. When \( \gamma = 1 \), the distribution becomes the Weibull, when \( \theta = 1 \) and \( \gamma = 1 \), the exponential, and as \( \gamma \to \infty \), the lognormal (Collett, 2014).

\subsection{2.5 Bayesian Survival Analysis}

By integrating prior information about the parameters, a posterior distribution for the parameters can be obtained and inferences on the model parameters and their functions can be made. Suppose \( \theta \) is some quantity that is unknown and let \( p(\theta) \) denote the prior distribution of \( \theta \). Next, let \( y \) be some observed
data whose probability of occurrence is assumed to depend on $\theta$. This dependence is formalized by $p(y \mid \theta)$, the conditional probability of $y$ for each possible value of $\theta$, and when considered as a function of $\theta$, is known as the likelihood ($L(\theta \mid y)$).

To obtain the posterior distribution, $p(\theta \mid y)$, the probability distribution of the parameters once the data have been observed, we apply Bayes’ theorem:

$$
p(\theta \mid y) = \frac{p(y \mid \theta)p(\theta)}{p(y)} = \frac{L(\theta \mid y)p(\theta)}{p(y)}
$$

Here, $p(y) = \int L(\theta \mid y)p(\theta)d\theta$ is a normalizing factor (constant). Thus, we have:

$$
p(\theta \mid y) \propto L(\theta \mid y)p(\theta)
$$

which simply means that the posterior distribution is proportional to the product of the likelihood and the prior. Prior distributions play a very important role in Bayesian statistics. There are two different types of prior distributions: informative and non-informative. In a parametric model, the distribution of outcomes (time to death) is specified in terms of a finite number of unknown parameters.

The likelihood function of the set of unknown parameters ($\theta$) in the presence of right censoring (for our data set) can be written as:

$$
L(\theta) = \prod_{j=1}^{n} f(t_j \mid X_j, \theta)^{\delta_j} \ast S(t_j \mid X_j, \theta)^{1-\delta_j}
$$

The log-likelihood form can be written as:

$$
\ell(\theta) = \sum_{j=1}^{n} \left[ \log \left( f(t_j \mid X_j, \theta)^{\delta_j} \right) + \log \left( S(t_j \mid X_j, \theta)^{1-\delta_j} \right) \right]
$$

where $f(t_j \mid X_j, \theta)$ and $S(t_j \mid X_j, \theta)$ are the density and survival distributions, respectively. The Bayesian AFT model for log-logistic and log-normal models can be obtained by the assumption that $\mu_j = X_j^T \beta$ (Christensen et al., 2011). In these models, when both $\beta$ and $\sigma^2$ are unknown, no joint
conjugate prior is available. A typical joint prior specification can be expressed as a product of normal (for the parameter $\beta | \sigma^2$) and an inverse gamma prior (for $\sigma^2$), that is:

$$\beta | \sigma^2 \sim N(\mu_0, \sigma^2 V_0) \text{ and } \sigma^2 \sim IG(\alpha, b)$$

The posterior distribution for the model specified above does not have closed form solutions for the parameters. Markov Chain Monte Carlo (MCMC) techniques can be used to sample from the joint posterior distribution of these models. In the present study, Metropolis-Hastings (MH) approach has been considered. Combining the likelihood function with the prior distributions on ($\beta, \sigma^2$) in the above models, the full conditional distributions for the unknown parameters are given by:

$$\pi(\beta | \sigma^2, t, X) \propto \prod_{j=1}^n f(t_j | X_j, \theta)^{\delta_i} \ast S(t_j | X_j, \theta)^{1-\delta_i} \ast \pi(\beta | \sigma^2) \ast \pi(\sigma^2)$$

For the log-normal distribution, for example, the density and survival distributions are given by:

$$f(t | X, \beta, \tau) = \frac{\sqrt{\tau}}{t \sqrt{2\pi}} \exp\left(-\frac{\tau}{2} \left[\log(t) - X'\beta\right]^2\right)$$

$$S(t) = 1 - \phi\left(\frac{\log(t) - X'\beta}{\sigma}\right), \ t > 0$$

3. Results

3.1 Descriptive statistics

A retrospective analysis was performed on 320 HIV infected patients who were under ART follow-up from 2008/9 to 2014/15 in Alamata General Hospital. Table 1 presents a summary of the demographic and baseline characteristics of the patients considered in this study.

Among the sampled HIV/AIDS patients who attended ART in Alamata General Hospital, 167 (52.18%) were male. When we consider the baseline characteristics of patients, 174 (54.37%) were unemployed and about half of them (48.75%) were in the age group 31 – 44. The level of education of patients was almost equally split between illiterate (no education), primary education and secondary & above education. Moreover, about half of the sampled patients (51%) had experienced past opportunistic infection.
Regarding baseline WHO clinical stage, 37 (11.6%), 45 (14.1%), 167 (52.2%) and 71 (22.2%) of the patients were in WHO stages I, II, III and IV, respectively. Among the patients in WHO stages I, II, III and IV at baseline, 8.1%, 15.6%, 22.8% and 57.7% have died during the study period, respectively. This shows that most of the patients that start ART in WHO stage-III and stage-IV had less survival time.

Table 1: Summary of baseline characteristics of HIV/AIDS patients

<table>
<thead>
<tr>
<th>Variables</th>
<th>Categories</th>
<th>Events/Failures</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Censored</td>
<td>Uncensored</td>
</tr>
<tr>
<td>Sex</td>
<td>Male</td>
<td>110</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>121</td>
<td>32</td>
</tr>
<tr>
<td>Age</td>
<td>15 - 30</td>
<td>87</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>31 - 44</td>
<td>111</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>≥ 45</td>
<td>33</td>
<td>32</td>
</tr>
<tr>
<td>Marital status</td>
<td>Never married</td>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Married</td>
<td>91</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>100</td>
<td>51</td>
</tr>
<tr>
<td>Level of education</td>
<td>No education</td>
<td>81</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>75</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Secondary and above</td>
<td>75</td>
<td>34</td>
</tr>
<tr>
<td>Employment</td>
<td>Working full-time</td>
<td>43</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Working part-time</td>
<td>44</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Not working due to illness</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Unemployed</td>
<td>120</td>
<td>44</td>
</tr>
<tr>
<td>Occupation</td>
<td>Farmer</td>
<td>120</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Civil servant</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Merchant</td>
<td>51</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Residence</td>
<td>Rural</td>
<td>107</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>Urban</td>
<td>124</td>
<td>43</td>
</tr>
<tr>
<td>TB screen</td>
<td>No</td>
<td>168</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>63</td>
<td>66</td>
</tr>
<tr>
<td>Past opportunistic infection</td>
<td>No</td>
<td>129</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>102</td>
<td>61</td>
</tr>
<tr>
<td>WHO clinical stage</td>
<td>I</td>
<td>34</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>38</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>129</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>30</td>
<td>41</td>
</tr>
</tbody>
</table>
The median baseline body weight of the HIV/AIDS patients was 50 kg and the average baseline CD4 cells count for HIV/AIDS patients was 126.01 (Table 2). We can also observe that the mean CD4 cell counts have consistently increased in subsequent years. This indicates that patients who were under ART follow-up have improved health status. The coefficient of variation also tells us that the variation in the CD4 cell counts among patients has consistently decreased from the baseline period to 2010/11.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Std. deviation</th>
<th>Coe. of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline weight (Kg)</td>
<td>51.68</td>
<td>50.0</td>
<td>10.37</td>
<td>20.1</td>
</tr>
<tr>
<td>Baseline CD4 cells</td>
<td>126.01</td>
<td>112.5</td>
<td>92.20</td>
<td>73.2</td>
</tr>
<tr>
<td>Average CD4 cells in 2008/9</td>
<td>164.00</td>
<td>142.5</td>
<td>118.76</td>
<td>72.4</td>
</tr>
<tr>
<td>Average CD4 cells in 2009/10</td>
<td>215.14</td>
<td>205.0</td>
<td>124.96</td>
<td>58.1</td>
</tr>
<tr>
<td>Average CD4 cells in 2010/11</td>
<td>305.74</td>
<td>289.0</td>
<td>164.56</td>
<td>53.8</td>
</tr>
</tbody>
</table>

### 3.2 Comparison of mean survival times of patients

Table 3 presents the results of the log-rank and Breslow tests that have been used for comparison of mean survival times of patients with respect to various categories of covariates. Both tests confirm the presence of a significant difference in the mean survival times of male and female patients under ART, that is, female patients had a significantly higher survival time compared to their male counterparts. There was also a significant difference in the mean survival time of patients in different age groups (log-rank test: p-value = 0.006 and Breslow test: p-value = 0.004). The results indicate that the mean survival time of patients in the age group 15 – 30 was greater than that of patients in the other age groups.

Among patients in different employment categories, patients who had no job because of illness had the lowest survive time as evidenced by the log-rank (p-value < 0.001) and Breslow (p-value < 0.001) tests. The tests also revealed that there was a significant difference in mean survival time of patients under different WHO clinical stages (log-rank test: p-value < 0.001 and Breslow test: p-value < 0.001). We can observe from the results that the mean survival time of patients in WHO stage IV was lower than that of patients in the remaining three categories.

Moreover, patients who were bedridden, exposed to opportunistic infections and diagnosed with TB had significantly shorter mean survival times. On the other hand, marital status, level of education and place of residence had no significant effect on the mean survival time of patients under ART.
Table 3: Comparison of survival time of HIV/AIDS patients under HAART

<table>
<thead>
<tr>
<th>Variables</th>
<th>Categories</th>
<th>Mean Survival</th>
<th>Log-Rank</th>
<th>Breslow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chi-sq</td>
<td>P-value</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chi-sq</td>
<td>P-value</td>
</tr>
<tr>
<td>Baseline Functional</td>
<td>Ambulatory</td>
<td>66.512</td>
<td>35.960</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td>Status</td>
<td>Bedridden</td>
<td>46.699</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Work</td>
<td>70.132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TB Screen</td>
<td>No</td>
<td>77.124</td>
<td>77.795</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>48.639</td>
<td></td>
<td>82.500</td>
</tr>
<tr>
<td>Opportunistic infection</td>
<td>No</td>
<td>73.409</td>
<td>18.808</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>58.493</td>
<td></td>
<td>20.800</td>
</tr>
<tr>
<td>WHO Clinical Stage</td>
<td>WHO stage-I</td>
<td>77.826</td>
<td>55.152</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>WHO stage-II</td>
<td>74.298</td>
<td></td>
<td>57.777</td>
</tr>
<tr>
<td></td>
<td>WHO stage-III</td>
<td>69.730</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WHO stage-IV</td>
<td>45.710</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sex</td>
<td>Male</td>
<td>61.422</td>
<td>7.471</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>70.698</td>
<td></td>
<td>8.178</td>
</tr>
<tr>
<td>Marital Status</td>
<td>Never Married</td>
<td>68.717</td>
<td>5.282</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>Married</td>
<td>69.043</td>
<td></td>
<td>4.482</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>62.370</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level of Education</td>
<td>No Education</td>
<td>69.680</td>
<td>3.605</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>65.372</td>
<td></td>
<td>3.366</td>
</tr>
<tr>
<td></td>
<td>Secondary &amp; above</td>
<td>62.442</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>Working Full Time</td>
<td>68.514</td>
<td>41.088</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>Working Part Time</td>
<td>73.310</td>
<td></td>
<td>38.039</td>
</tr>
<tr>
<td></td>
<td>Not Working due to illness</td>
<td>40.290</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unemployed</td>
<td>68.267</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispense code</td>
<td>No</td>
<td>65.958</td>
<td>5.019</td>
<td>0.170</td>
</tr>
<tr>
<td>Contrimoxazole</td>
<td>Yes</td>
<td>64.547</td>
<td></td>
<td>4.595</td>
</tr>
<tr>
<td>Occupation</td>
<td>Farmer</td>
<td>66.010</td>
<td>0.012</td>
<td>0.914</td>
</tr>
<tr>
<td></td>
<td>Government Employee</td>
<td>29.009</td>
<td></td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>Merchant</td>
<td>67.991</td>
<td>8.153</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>73.743</td>
<td></td>
<td>8.512</td>
</tr>
<tr>
<td>Number of living rooms</td>
<td>1</td>
<td>64.084</td>
<td>4.406</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>70.329</td>
<td></td>
<td>3.022</td>
</tr>
<tr>
<td></td>
<td>3 and above</td>
<td>68.286</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residence</td>
<td>Rural</td>
<td>64.714</td>
<td>0.604</td>
<td>0.437</td>
</tr>
<tr>
<td></td>
<td>Urban</td>
<td>66.964</td>
<td></td>
<td>0.487</td>
</tr>
<tr>
<td>Age</td>
<td>15 - 30</td>
<td>76.150</td>
<td>25.710</td>
<td>p &lt; 0.001</td>
</tr>
<tr>
<td></td>
<td>31 - 44</td>
<td>64.800</td>
<td></td>
<td>25.030</td>
</tr>
<tr>
<td></td>
<td>≥ 45</td>
<td>53.520</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to Kaplan-Meier survival estimates for the two sex groups (Figure 1), female patients had higher survival time compared to men. This result is supported by the log-rank and Breslow tests shown in Table 3. Comparison of Kaplan-Meier survival curves with respect to WHO clinical stages also shows that patients in baseline WHO clinical stage IV had the lowest survival time (Figure 2).
Figure 1: Kaplan-Meier survival curves for HIV infected patients under HAART by sex

Figure 2: Kaplan-Meier survival curves for HIV patients by baseline WHO clinical stage

3.3 Comparison of parametric survival models

Cox-Snell residual plot and QQ plot as well as AIC and log-likelihood statistics were used to identify the appropriate parametric survival model among the six commonly considered survival models (Avci, 2017; Khanal et al., 2014). From the results in Table 4 we can observe that the log-normal survival model has the smallest AIC and log-likelihood statistic. This indicates that the log-normal survival model is a better-fit to the data as compared to the other parametric survival models.

Table 4: Comparison of parametric survival models

<table>
<thead>
<tr>
<th>Model</th>
<th>DF</th>
<th>AIC</th>
<th>Log-likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gompertz</td>
<td>20</td>
<td>991.1</td>
<td>-475.6</td>
</tr>
<tr>
<td>Exponential</td>
<td>18</td>
<td>990.2</td>
<td>-476.1</td>
</tr>
<tr>
<td>Weibull</td>
<td>18</td>
<td>986.6</td>
<td>-473.3</td>
</tr>
<tr>
<td>Generalized Gamma</td>
<td>21</td>
<td>985.8</td>
<td>-472.9</td>
</tr>
<tr>
<td>Log-logistic</td>
<td>18</td>
<td>984.9</td>
<td>-475.5</td>
</tr>
<tr>
<td>Log-normal</td>
<td>18</td>
<td>980.8</td>
<td>-472.4</td>
</tr>
</tbody>
</table>
It is common practice to use Cox-Snell residuals to check for overall goodness of fit in survival models. If a model is a good fit to the data, we expect the Cox-Snell residuals to cluster around a straight line through the origin with unit slope. We can see from the Cox-Snell residual plots in Figure 3 that the log-normal survival model is a better fit to our data.

Figure 3: Cox-Snell residual plots for parametric survival models
A Quantile-Quantile plot (or Q-Q plot) is another graphical method that is commonly used to check whether a survival model provides an adequate fit to data or not. In this study, the adequacy of the accelerated failure-time model was investigated by comparing two age groups (HIV infected patients in the age group 30 – 44 and above 45 years) and two functional status groups (patients in bedridden group and in working group). The Q-Q plots are shown in Figure 4. The plots approximate a 45 degree straight line through the origin indicating that the log-normal accelerated failure time model is an appropriate model.

![Quantile-Quantile plots](image)

(a) q-q plot for age group 
(b) q-q plot for functional status group

Figure 4: Quantile-Quantile plots for selected covariates

Thus, this study identified and analyzed the predictors of HIV patients’ survival based on Bayesian approaches through the log-normal distribution.

### 3.4 Bayesian Analysis

In the Bayesian analysis, MCMC sampling algorithm was implemented with 40000 iterations in three different chains, and 20000 burn-in terms were discarded from each so as to get 60000 samples from the posterior distribution. Convergence diagnosis was made through trace (time series), density and autocorrelation plots of parameter estimates. All diagnostic measures suggested that each of the parameter estimates has converged to its posterior distribution. Table 5 presents the results of the fitted Bayesian log-normal AFT model based on the sample obtained from the joint posterior distribution.

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Acceleration Factor</th>
<th>95% CrI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age ≥ 45</td>
<td>0.8787</td>
<td>0.7700 – 0.9825</td>
</tr>
</tbody>
</table>

The covariates with a credible interval that does not include zero are significant. The acceleration factor of each of the significant variables in Bayesian log-normal model is interpreted as in the case of classical model. The acceleration factor for covariate age ≥ 45 is $\varphi = 0.8787$ (95% CrI: 0.7700 – 0.9825). This shows that the survival time of HIV infected patients who were in the age group ≥ 45 was about 12%
lower relative to patients in the age group 15 – 30, keeping the other variables constant. The acceleration factor of $\varphi = 1.0200$ for weight indicates that the survival time of HIV patients was increased by a factor of 1.02 as the weight of the patient increased by one kilogram. The acceleration factors for bedridden and work functional status are $\varphi = 0.4075$ (95% CrI: 0.2286 – 0.7138) and $\varphi = 1.4349$ (95% CrI: 1.0075 – 2.5449), respectively. This implies that the survival time of HIV infected patients under HAART who were actively working was higher by a factor of 1.4349 relative to the reference category (ambulatory group). Moreover, the survival time of bedridden HIV infected patients was about 59% lower compared to those in the ambulatory group, keeping the other variables constant.

Concerning the sex covariate, the acceleration factor of $\varphi = 1.7794$ (95% CrI: 1.2236– 2.5852) for female patients indicates that the survival time of female patients was higher by a factor of 1.7794 compared to male HIV infected patients. The covariates baseline CD4 count, TB screen and WHO clinical stage are also significant.

Table 5: Bayesian posterior summary of parameter estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean ($\hat{\beta}$)</th>
<th>$\varphi = \exp(\hat{\beta})$</th>
<th>MC-error</th>
<th>95% credible interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower</td>
</tr>
<tr>
<td>Age (≥ 45)</td>
<td>-0.1293</td>
<td>0.8787</td>
<td>0.003601</td>
<td>-0.2614</td>
</tr>
<tr>
<td>Functional status (bedridden)</td>
<td>-0.8976</td>
<td>0.4075</td>
<td>0.002598</td>
<td>-1.4760</td>
</tr>
<tr>
<td>Functional status (work)</td>
<td>0.3611</td>
<td>1.4349</td>
<td>0.002665</td>
<td>0.0075</td>
</tr>
<tr>
<td>TB screen (positive)</td>
<td>-1.6490</td>
<td>0.1922</td>
<td>0.002642</td>
<td>-2.1050</td>
</tr>
<tr>
<td>Past opportunistic infection (Yes)</td>
<td>-0.8774</td>
<td>0.4159</td>
<td>0.002385</td>
<td>-1.3140</td>
</tr>
<tr>
<td>Baseline CD4 cells</td>
<td>0.0021</td>
<td>1.0020</td>
<td>0.000000</td>
<td>0.0001</td>
</tr>
<tr>
<td>WHO clinical stage (I)</td>
<td>0.8320</td>
<td>2.2980</td>
<td>0.005233</td>
<td>0.0844</td>
</tr>
<tr>
<td>Sex (female)</td>
<td>0.5763</td>
<td>1.7794</td>
<td>0.001593</td>
<td>0.2018</td>
</tr>
<tr>
<td>Marital status (others)</td>
<td>-0.4236</td>
<td>0.6547</td>
<td>0.005544</td>
<td>-1.1900</td>
</tr>
<tr>
<td>Employment (1)</td>
<td>0.9545</td>
<td>2.5974</td>
<td>0.005087</td>
<td>0.1553</td>
</tr>
<tr>
<td>Employment (3)</td>
<td>0.7042</td>
<td>2.0220</td>
<td>0.006456</td>
<td>0.1115</td>
</tr>
<tr>
<td>Dispense (3)</td>
<td>4.8330</td>
<td>125.5870</td>
<td>0.010140</td>
<td>2.6690</td>
</tr>
<tr>
<td>Occupation (1)</td>
<td>0.7577</td>
<td>2.1330</td>
<td>0.003085</td>
<td>0.1492</td>
</tr>
<tr>
<td>Occupation (3)</td>
<td>2.1660</td>
<td>8.7233</td>
<td>0.004160</td>
<td>1.4040</td>
</tr>
<tr>
<td>Number of rooms (3)</td>
<td>1.7340</td>
<td>5.6630</td>
<td>0.003858</td>
<td>0.7437</td>
</tr>
<tr>
<td>Baseline weight</td>
<td>0.0200</td>
<td>1.0200</td>
<td>0.000400</td>
<td>0.0017</td>
</tr>
<tr>
<td>Sigma</td>
<td>79.60</td>
<td>0.006310</td>
<td>78.08</td>
<td>80.38</td>
</tr>
</tbody>
</table>

### 3.5 Discussions

Descriptive statistics results revealed that there was a good immune recovery after ART follow-up, that is, the mean baseline CD4 cells count of HIV/AIDS patients has increased from 126.01 to 305.74 after three
years of follow-up. This result is consistent with those of Derbe et al. (2013) and Holla et al. (2016). Generally, death of HIV/AIDS patients was higher among those in WHO clinical stage-III (22.75%) and IV (57.75%). A relatively low proportion of female HIV patients under HAART (20.92%) have experienced the event compared to male patients (34.13%). This result is in line with a similar study conducted by Grover et al. (2013). Moreover, a relatively higher percentage of divorced or widowed patients have died compared to those who were never married and married (33.77% versus 23.08% and 22.22%). This result is not in line with Grover et al. (2013).

The results from the fitted Bayesian survival models revealed that the survival time of HIV patients has increased with an increase in the CD4 cell counts. This finding is similar to the studies conducted by Grover et al. (2013) and Tigist et al. (2012). The survival time of HIV infected patients who were in the age group $\geq 45$ was lower than those in age group 15 – 30. The results also indicated that the survival time of male patients, patients in WHO clinical stage IV, patients with past opportunistic infection, bedridden patients, patients not working because of illness and TB positive HIV patients was lower compared to that of patients in respective reference categories. These results are consistent with the findings of a number of studies (Grover et al., 2013; Makombe et al., 2007; Holla et al., 2016; Chalachew et al., 2017; Seid et al., 2014). Among the covariates considered in this study, level of education and residence were not significant.

4. Conclusion

Among the parametric survival models considered in this study, log-normal AFT model was found to be a better-fit for the HIV infected patients dataset through various graphical and non-graphical diagnostic methods (AIC, log-likelihood, Q-Q plot and Cox-Snell residuals plot). The results from the fitted model revealed that baseline weight, baseline functional status, TB screen, baseline CD4 cell counts, baseline WHO clinical stage, marital status, occupation and employment status were prognostic factors exhibiting considerable association with survival time of HIV/AIDS patients under ART follow-up in the study area. Thus, potential stakeholders, policy makers and clinicians should give due attention to the identified potential factors that may influence patients’ survival in a bid to prolong the life of HIV patients.

References


Counting Process Conditional Modeling of Recurrence of Dropout of School Students: The Case of Hawassa City & Hawassa Zurya Woreda, SNNPR, Ethiopia

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² Kotebe Metropolitan University, Addis Ababa, Ethiopia

Abstract

Primary school dropout is a serious problem in many low and middle income countries since it limits the future opportunities for children and is a waste of limited resources. This paper aims to model and investigate the recurrence behavior of school dropout of primary students using the counting process conditional modeling approach. School students commonly experience recurrence of dropouts and it is very crucial to investigate the effects of covariates on such recurrence of school dropout. A total of 1360 school students were followed up between the years 2008 (start) to 2015 (end of study period) in 21 rural and urban schools. The counting process conditional model with the gap between consecutive occurrence times was used to fit the data. A maximum of four recurrences was observed during this follow-up period. Frequent school absenteeism, high peer pressure to dropout, and bad living condition were found out to increase the risk of recurrence of school dropout at student level. At family level, parental demand for child work and, at school level, school location in rural areas and long distance to school, increased the risk of recurrence of school dropout. Moreover, high student motivation score, high family income, high parental educational level, high teacher & school characteristic scores, and relevance of education system score decreased the hazard rate of dropout recurrence at the primary schools. In summary, the counting process conditional model was found helpful in modeling the recurrence nature of school dropout. The risk factors identified in this study can be used for interventions to reduce recurrence of school dropout in Ethiopia.

Keywords: Counting Process Conditional model, recurrent event, School Dropout

1. Introduction

High dropout of students at primary education has attracted the attention of researchers in low and middle income developing countries. Primary school dropout is an important issue in developing countries since it results in waste of public money and reduces the proportion of students who complete primary school, and consequently, compromises qualified human resources. Children may dropout from school
permanently or temporarily due to various reasons. Different studies pointed out that these reasons are country specific depending on social, political, cultural and economic circumstances. Joubish & Khurram (2011) showed that low educational level of parents; poverty (as students cannot afford to go to schools); children’s lack of interest towards education; demand for child labor; punishments at school; teacher qualities; and adverse school environment were factors contributing to dropout at primary level.

Jamil et al. (2010) indicated that poverty in the family is one of the main causes for the drop out of students at primary level in rural areas. The same study revealed that school distance, large family size, overcrowded classrooms, punishment and school fees were also factors for dropout of students. According to Chistle et al. (2007), students’ ethnical background, availability of job opportunities after education, climate of schools and parents perception towards education influence the enrollment and drop out of children from schools.

Studies by Plank et al. (2005) and Entwisle et al. (2005) suggested that being over-aged in a grade significantly increases the hazard of school dropout of students and overshadows academic achievement. Rumberger (2004) stated that lack of engagement (typically measured by school absenteeism) and student behavior in primary schools predicted dropout from school. Appleton et al. (2008) has noted that the effects of psychological and cognitive types of engagement, including low motivation and psychological or behavioral problems like anxiety and disciplinary problems, result in dropout from school.

Dalton et al. (2009) reported that socioeconomic status of parents often measured by parents’ occupational status, educational level and income are predictors of school dropout of students. Many scholars stress the importance of parental income on school dropout of students and found that low family income increases the likelihood of dropout (e.g., Ishitani and Snider, 2006; Ou and Reynolds, 2006; Cataldi et al., 2009). Furthermore, other studies reported that students from large families (with five or more siblings) and from single-parent families are more likely to dropout of school (Bridgeland et al., 2006 and Dustmann and Van Soest, 2007). In their study, Cooper et al. (2005) and Ishitani and Snider (2006) stated that parental involvement is a predictor of school dropout. Parental involvement is perhaps the most important family factor that scholars have agreed upon, irrespectively of family income.

Researchers in education have contended that smaller class sizes are likely to result in lower rates of school dropout and its effect is related to schools’ social climate. Moreover, teachers’ experience and expectations (Dalton et al., 2009), support and instruction quality (Blue and Cook, 2004) and teacher quality (Bridgeland et al., 2006) have been found out to influence dropout.
According to UNESCO (2014) report, “despite improvements in enrollment of children into school, dropout before grade completion remains a serious problem in many low and middle income countries”. In Ethiopia, primary school dropout rate has almost doubled from 9% in 1999/00 to 19% in 2003/04, and then declined to 12% in 2004/05. This figure has increased to 12.4%, 14.6% and 18.6% in 2006/07, 2007/08 and 2008/09, respectively. The rate has fluctuated substantially in subsequent years and declined to 15.7% in 2012/13. These figures, coupled with primary completion rate of just 52.8% in 2012/13, are an indication that higher primary school dropout and lower completion rates are serious problems in Ethiopia (MoE, 2013).

Despite its importance, strategies designed to improve primary school progression and retention has been given little attention. Children enroll into primary schools in greater numbers but there is massive dropout which hampers progression into higher grades. We are therefore still far in achieving universal education for all. As a result of substantial rates of dropout and non-completion of primary school, many children are leaving schooling without acquiring the basic knowledge. This limits not only future opportunities for children, but is also a waste of limited resources of the country. Almost no previous researchers of education considered the recurrence behavior of primary school dropout. This paper has focused on recurrence nature of school dropout using the generalized Cox regression (conditional counting process) model using seven years of primary school data spanning from 2008 to 2015.

Survival models have been extensively used in health and economics, and are witnessing expanded use across the social sciences and education to explicitly model the dynamic nature of educational process (Plank at el, 2008). This is certainly helpful in modeling the dynamic pattern of student school dropout (Willett and Singer, 1991). Event-history modeling is well suited to study the recurrence nature of student school dropout and allows to incorporate variables that capture this changing circumstances faced by students.

Repeated event processes are such that individuals experience the same or different types of events more than once over time. In many scientific investigations, the outcome variable of interest is a recurrent event. School dropout is often recurrent in that some students experience more than one dropout and subsequent dropouts influenced by previous occurrences. The data structure of recurrence of school dropout is naturally the ordered occurrences in time of the event and often the recurrence of events ‘within” an individual are correlated.
Numerous statistical models have been proposed to analyze recurrent event data. Focus can usually be placed on time-between-event (i.e., gap times or inter-occurrence time). In the recurrent event history data literature, the generalized Cox PH model or counting process models are popular, allowing all of the events for each individual to be analyzed. Counting process conditional or Prentice, Williams and Peterson (PWP) model is one of the prominent regression models for analyzing recurrent event data. This model preserves the sequential order of school dropout to define the risk set of each event, incorporates event dependence and allows estimating the model in gap time layout. To our knowledge, this model has not been tested on school dropout history data. This study aims to model the recurrence of primary school dropout of students with the generalized Cox regression (conditional counting process) model and investigate the effect of covariates related to students, families, teachers and schools.

2. Primary school student cohort data

We considered children who started schooling in September 2008 with various ages (from seven years of age, which corresponds to normal schooling start age, up to 13 years) in Ethiopia. We reconstructed their schooling path until June 2015. Pupil may have dropped out of school permanently or temporarily. In the latter case, students may re-enter back to school after dropping out and repeat different grade levels.

The duration of the event for each of the students in the study was defined as the time between enrolment and the end of the study period or until the subject dropped out. During the follow up period, the status of a student is whether he/she has dropped out from class by the end of each academic year or not. The outcome variable was coded as 1 if a student experiences dropout at the end of an academic year during the follow up period and 0 otherwise.

A distinguishing feature of the data for the generalized Cox regression models is the data layout and the risk set. The data layouts are organized as lists of start time and stop time for each interval of follow-up corresponding to the number recurrent dropouts stratified by the number of events (Therneau & Grambsch, 2000; Prentice et al. 1981; Kelly & Lim, 2000; Fleming & Harrington, 1991; and Castaneda and Gerritse, 2010). One example of dropout history data is plotted in Figure 1 for illustration. The figure represents a data layout structure having different start and stop times with two recurrent events. This student (72th sample) experienced the event of interest (dropout) two times at times 2 and 5 (7 corresponds to censoring time due to end of study period). The data layout has 3 periods: [0, 2], [2, 5] and [5, 7] representing the data structure for the counting process modeling.
3. Statistical models

3.1 Counting process conditional (PWP) Models

Prentice et al. (1981) proposed a generalized Cox PH which is known as the counting process conditional (or PWP) model to analyze recurrent events. The model extends the classical survival model to a more general multiple event model. It is very flexible in the formation of strata and risk sets, definition of the time scale, and estimation of effects of covariates (Fleming & Harrington, 1991, Therneau & Grambsch, 2000; Andersen & Gill, 1982; Kelly & Lim, 2000). The PWP model handles ordered multiple events by stratification based on the number of events during the follow-up period to account for the dependence among events within subjects. The use of time dependence in strata in the underlying hazard function can vary from event to event and accounts for correlation of recurrent events.

Let \((T_j, C_j, X_j)\) denote the failure time, censoring time, and the covariates for \(j^{th}\) event, respectively. The covariates are of dimension \(p\) and may be time dependent. Let \(N_i(t) = \int_0^t dN_i(s)\) denote the number of events occurring in time interval \([0, t]\) for the \(i^{th}\) subject, where \(dN_i(t)\) denotes the number of events (increment) in a small time interval \([t, t+\Delta t]\). Then \(N_i(t)\) is a counting process for the recurrent event with the following properties (Cook and Lawless, 2010):

i) \(N_i(t) \geq 0\)

ii) \(N_i(t)\) is integer valued

iii) for \(s < t\), \(N_i(s) \leq N_i(t)\)

iv) for \(s < t\), the number of events that occurred in the interval \((s, t)\) is given by \(N_i(t) - N_i(s)\)
v) \( P\{N_i(t + \Delta t) - N_i(t) \geq 2\} = o(\Delta t) \)

Given the risk indicator function, \( Y_i(t) = I(T_i \geq t) \), the full information of event history of the stochastic process for the \( i^{th} \) subject has the form:

\[
H_i(s) = \{X_i(t); Y_i(t); N_i(t) \geq 0 \leq t \leq s\} \quad \text{................................................... (1)}
\]

where \( X_i(s) = \{X(t) : t < s\} \) and \( Y_i(s) = \{Y(t) : t < s\} \) represent the process history up to and including time \( t \) (Cook and Lawless, 2010).

Under this counting process, the conditional intensity function for event history has the form:

\[
\lambda_i(t \mid H_i(t)) = \lim_{\Delta t \to 0} \frac{\text{prob} \{N_i(t + \Delta t) - N_i(t) = 1 \mid H_i(t)\}}{\Delta t} \quad \text{.............................................. (2)}
\]

It is assumed that the probability of occurrence of more than one event in the interval \([t, t + \Delta t]\) is in order of \( o(\Delta t) \) so that \( \mathbb{E}[dN_i(t) \mid H_i(t)] = \lambda_i(t; H_i(t)dt) \). Then the counting process conditional intensity function for the \( j^{th} \) recurrence is connected to the covariates through the following formulation:

\[
\lambda_{ij}(t, H(t)) = Y_{ij}(t)\lambda_{ij0}(t)\exp(\beta'X_{ij}(t)) \quad \text{.......................... (3)}
\]

The corresponding survival function for the above model is given as:

\[
S_{ij}(t) = \exp(-\Lambda(t)Y_{ij}(t)\exp(\beta'X_{ij}(t))) \quad \text{......................... (4)}
\]

where \( \Lambda(\cdot) \) is the cumulative baseline hazard function. This model can be considered as a stratified counting process which provides event-specific baseline hazards and risk set as well as unbiased estimates of overall and event specific covariates effect (Kelly & Lim, 2000; Therneau & Grambsch, 2000).

The estimation of parameters is based on partial likelihood (Prentice at el., 1981; Cox, 1972). The log partial likelihood has the form:

\[
\log L(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{J} \delta_{ij}(t) \left[ \beta'X_{ij}(t) - \log \left( \sum_{i=1}^{n} \sum_{m=1}^{J} Y_{im}(t) \exp \left\{ \beta'X_{ij}(t) \right\} \right) \right] \quad \text{.......................... (5)}
\]

The corresponding score equation is expressed as:

\[
U(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{J} \delta_{ij}(t) \left[ X_{ij}(t) - \frac{\sum_{i=1}^{n} \sum_{j=1}^{J} Y_{ij}(t) \exp \left\{ \beta'X_{ij}(t) \right\} X_{ij}(t)}{\sum_{i=1}^{n} \sum_{j=1}^{J} Y_{ij}(t) \exp \left\{ \beta'X_{ij}(t) \right\}} \right] \quad \text{.......................... (6)}
\]
where $\delta_i(t)$ denotes the increment in event over the time interval $[t, t+\Delta t]$. The maximum partial likelihood estimator ($\hat{\beta}$) is obtained by equating $U(\beta) = 0$ and solving for the unknowns using iterative methods. The estimator is asymptotically normally distributed with mean $\beta$ and variance $I^{-1}(\hat{\beta})$. The robust variance estimator is based on a sandwich estimate given by $V = I^{-1}(\beta)B I^{-1}(\beta)$, where $I^{-1}(\beta)$ is the inverse of the information matrix and $B$ is a correction factor for survival setting and overcomes the dependence of multiple events per subject to correct standard errors or adjust correlation (Cas-taneda & Gerritse, 2010; Kelly & Lim, 2000; Therneau & Grambsch, 2000; Therneau & Hamilton, 1997). In this study the R Survival package was used to estimate the parameters, and to check the proportional hazards assumption or goodness-of-fit.

The PWP model handles both event specific covariate effect ($\beta_k$) and a weighted average effect ($\beta$) by imposing the restriction $\beta_k = \beta$ during maximization of partial likelihood. It allows the baseline hazard function to vary over each of the separate events. In practice, data may need to be limited to a specific number of recurrent events if the risk set becomes very small for latter strata making event-specific estimates unreliable. Due to this fact, the parameters estimated are the overall or weighted average of each stratum effect of the covariate in this study.

The PWP model can also be defined in terms of gap time (GT), which is the time since the previous event. When the time between events is the variable of interest, the occurrence of each recurrent event is considered as the time origin for the occurrence of the next event in the correlated failure time survival analysis (Castaneda and Gerritse, 2010; Kelly & Lim, 2000).

### 3.2 Survival estimation methods for recurrent events

Statistical methods of inference in the presence of recurrent events has been considered by several authors in fields of health sciences, engineering, social sciences, etc. to compare groups in recurrent survival analysis (e.g., Wang and Chang, 1999, Prentice, 1978 and Peña, et al, 2001). These are summarized as follows:

#### 3.2.1 Wang-Chang estimator (WCE)

Wang and Chang (1999) developed a non-parametric estimator of the survivor function when the within-unit inter-occurrence times are correlated for recurrent events. The WC estimator is given by:
\[ \hat{S}(t) = \prod_{i=1}^{n} \prod_{j:T_{ij} \leq t} \left( 1 - \frac{d^*(T_{ij})}{R^*(T_{ij})} \right) \] ........................... (7)

where \( T_{ij} \) denotes the inter-occurrence time of the \( j^{th} \) event for the \( i^{th} \) subject, \( R^*(t) \) represents the average number of individuals at risk at time \( t \) and \( d^*(t) \) denotes the sum of the weighted average of the total number of observed uncensored recurrent times for a subject that are equal to \( t \).

### 3.2.2 Generalized product limit estimator (GPLE)

Peña et al. (2001) proposed two recurrent event survival functions. One model is GPLE that is a generalization of the classical Kaplan-Meier estimator where the events are recurrent and independently and identically distributed inter-occurrence times. The counting process \( N(s, t) \) represents the number of observed events in the interval \([0, s]\) with \( T_{ij} \leq t \) and \( Y(s, t) \) represents the number of observed events in the interval \([0, s]\) with \( T_{ij} \geq t \). Then the GPLE estimator has the form:

\[ \hat{S}(t) = \prod_{w \leq t} \left[ 1 - \frac{\Delta N(s, w)}{Y(s, w)} \right] \] .......................... (8)

where \( \Delta N(s, w) = N(s, w + \Delta w) - N(s, w) \) and \( \Delta w \) is considered to be small interval of time.

The second estimator, which is known as the MLE for frailty model or FRMLE, uses a Gamma like distribution with shape and scale parameters equal to an unknown parameter \( \alpha \) and takes the form:

\[ \hat{S}(t) = \left[ \frac{\hat{\alpha}}{\hat{\alpha} + \hat{\Lambda}_0(t)} \right]^\hat{\alpha} \] ............................... (9)

where \( \hat{\Lambda}_0(t) \) is an estimator of the marginal cumulative hazard function \( \Lambda(t) \). The specific value for \( \alpha \) is inversely related to the amount of correlation that exists between recurrence times, that is, as \( \alpha \) increases, the correlation decreases so that letting \( \alpha \to \infty \) would imply complete independence.

### 4. Results and discussion

#### 4.1 Descriptive analysis

Table 1 presents a summary of recurrences of school dropout during the follow-up period. Out of 1360 primary school students, 53.5% of them have dropped out of school at least once. Specifically, 22.3%, 16.5%, 12.9% and 1.9% of them experienced one, two, three and four dropouts, respectively.
School participation was examined by two outcome variables to capture schooling decisions. The first is the age at which a child was enrolled in school. As many of the children were not enrolled at age seven (normal age), a child’s age of entry into school system was computed retrospectively using parents’ accounts of the year of schooling the child achieved by the age of 15. The second measure of participation was the highest grade attained by the child at the age of 15 which helps to capture the extent of primary school completion. Table 2 presents summary statistics of these two school participation variables cross-tabulated by sex and socio-economic background of families. The results show that there was vertical inequality of children’s schooling based on school location, gender and household background factors such as parents’ educational level, family income, and birth order of the child.

### Table 2: Summary statistics of school participation variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category</th>
<th>Age of entry to school</th>
<th>Highest grade completed by age 15</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>School location</td>
<td>Rural</td>
<td>8.85</td>
<td>1.086</td>
</tr>
<tr>
<td></td>
<td>Urban</td>
<td>7.27</td>
<td>1.185</td>
</tr>
<tr>
<td>Gender</td>
<td>Male</td>
<td>8.05</td>
<td>1.173</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>7.94</td>
<td>1.088</td>
</tr>
<tr>
<td>Father education</td>
<td>No education</td>
<td>8.52</td>
<td>1.166</td>
</tr>
<tr>
<td></td>
<td>Primary education</td>
<td>8.09</td>
<td>1.165</td>
</tr>
<tr>
<td></td>
<td>High School</td>
<td>7.81</td>
<td>1.048</td>
</tr>
<tr>
<td></td>
<td>Higher education</td>
<td>7.19</td>
<td>1.033</td>
</tr>
<tr>
<td>Mother education</td>
<td>No education</td>
<td>8.18</td>
<td>1.173</td>
</tr>
<tr>
<td></td>
<td>Primary education</td>
<td>7.97</td>
<td>1.120</td>
</tr>
<tr>
<td></td>
<td>High school</td>
<td>7.74</td>
<td>1.040</td>
</tr>
<tr>
<td></td>
<td>Higher education</td>
<td>7.76</td>
<td>1.037</td>
</tr>
<tr>
<td>Family income</td>
<td>Low</td>
<td>9.12</td>
<td>1.175</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>7.93</td>
<td>1.091</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>6.79</td>
<td>1.104</td>
</tr>
<tr>
<td>Birth order of student</td>
<td>1st</td>
<td>7.03</td>
<td>1.116</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>7.99</td>
<td>1.042</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>8.03</td>
<td>1.072</td>
</tr>
<tr>
<td></td>
<td>4th</td>
<td>8.18</td>
<td>1.165</td>
</tr>
<tr>
<td></td>
<td>5th</td>
<td>8.80</td>
<td>1.184</td>
</tr>
</tbody>
</table>

### 4.2 Estimation of confidence interval limit for the recurrent event

The Wang and Chang (WC) 95% confidence interval product limit estimates for the recurrence of school dropout is given in Figure 2. The WC non-parametric analysis revealed that the median survival time was...
five years with median of recurrence of two. The plot for the recurrent school dropout indicated that the probability of dropout declines with increasing time. For instance, by the time \( t = 7 \), the probability is approximately 0.39.

![Figure 2: Confidence interval limit of intercurrence times for recurrent dropout](image)

### 4.3 Comparison of group survival functions involving recurrent events

In this section we compare survival curves by school location (rural versus urban) and gender (male versus female). The analysis was carried out using the generalized product limit estimator (GPLE).

The results in Table 3 indicated that there is a significant difference in the survival probability of dropout among students attending schools in rural and urban localities in all tests. In the case of gender, some of the tests (Mrec and Grec) also indicate that there is a significant difference in the survival probability of dropout between male and female students.

<table>
<thead>
<tr>
<th>Method</th>
<th>Chi-square</th>
<th>p-value</th>
<th>Chi-square</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRrec</td>
<td>21.949</td>
<td>&lt;0.001</td>
<td>0.952</td>
<td>0.329</td>
</tr>
<tr>
<td>Grec</td>
<td>9.318</td>
<td>&lt;0.001</td>
<td>3.600</td>
<td>0.045</td>
</tr>
<tr>
<td>TWrec</td>
<td>14.692</td>
<td>&lt;0.001</td>
<td>2.362</td>
<td>0.124</td>
</tr>
<tr>
<td>PPrec</td>
<td>14.245</td>
<td>&lt;0.001</td>
<td>2.444</td>
<td>0.117</td>
</tr>
<tr>
<td>PMrec</td>
<td>14.237</td>
<td>&lt;0.001</td>
<td>2.446</td>
<td>0.117</td>
</tr>
<tr>
<td>PPrec</td>
<td>13.133</td>
<td>&lt;0.001</td>
<td>2.746</td>
<td>0.097</td>
</tr>
<tr>
<td>HFrec</td>
<td>27.254</td>
<td>&lt;0.001</td>
<td>0.952</td>
<td>0.329</td>
</tr>
<tr>
<td>CMrec</td>
<td>27.254</td>
<td>&lt;0.001</td>
<td>0.952</td>
<td>0.329</td>
</tr>
<tr>
<td>Mrec</td>
<td>5.976</td>
<td>0.015</td>
<td>4.668</td>
<td>0.030</td>
</tr>
</tbody>
</table>

To determine which recurrent event survival estimator yields the most accurate estimates for our data, the main consideration is dependence. Figure 3 displays the survival curves computed with Pena, Strawderman and Hollander (PSH), WC and FRMLE methods. Since the generalized Kaplan-Meier estimator (GPLE) assumes that survival times are independently and identically distributed, it might not
be the best choice given that the plots in Figure 3 showed signs of dependence. The other two estimators that do allow for dependence would rather be more appropriate.

The plot presented in Figure 4 shows that the hazard function has different effect on each recurrence of event times in event-wise analysis. The plot also suggests a relatively higher survival probability for the first recurrent event than the second and third recurrence times. However, there is no clear pattern of survival probability between second and third recurrent events times, suggesting that the risk of a new recurrent event does not remain constant. The plot also provides vital information about the model to be fitted: time varying strata based on the number of events. A model that encompasses this feature is the PWP model that makes use of time-dependent strata, that is, the underlying hazard function may vary from event to event to account for the dependence of events within subjects.
The results in Table 3 revealed that there are significant differences in the survival probability of dropouts depending on school location. However, the results for gender are inconclusive. Then it is important to check the variation in the recurrence pattern with respect to these covariates without adjusting for the other covariates. Figure 5 and 6 present the survival probabilities for the first four consecutive events by gender and school location. It can clearly be seen that the risk of recurrence of events does not remain constant with respect to school location. For instance, the survival probability is in general higher for students in urban localities as compared to those in rural schools, and the probabilities decline over years of schooling and increase with the number of recurrences. When we come to gender, on the other hand, the survival curves for male and female students almost overlap each other. However, females seem to have a relatively higher risk than males in the occurrence of the first dropout.

![Graphs showing survival probabilities for Consecutive recurrence events by gender](image)

**Figure 5:** Survival probabilities for consecutive recurrence events by gender
4.4 Test of proportionality assumption of hazard of counting process conditional model

The results of the global goodness-of-fit test of the model to the data, that is, whether the proportionality assumption in PWP is violated or not, indicated that the global chi-square test statistic was not significant. Thus, the assumption of proportionality of hazards of the generalized Cox model is not violated. Hence, we can proceed with the analysis using the counting process conditional model.

4.5 Results and discussion of the fitted PWP model

The results of the fitted counting process conditional model are displayed in Table 4. The covariates are classified into student level, family level and school level factors. The model was found to be a good-fit to the data as judged by the likelihood-ratio, Wald and score tests.

The results show that entry age, birth order of the pupil, time spent on study, student absenteeism, peer pressure, student ethics, and student motivation & living condition scores are significant covariates responsible for the risk of recurrence of dropout. The results indicated that increase in school entry age by one year increases the risk of recurrence of dropout by 23.9% (HR=1.239; 95% CI: 1.002 – 1.533). The more time a student spends on studying, the lower the risk of recurrence of dropout. Moreover, high
student motivation score decreases the risk of recurrence of dropout by 5.08% (HR= 0.492; 95% CI: 0.372 – 0.650), while frequent student absenteeism from school and low living condition score increase the risk of recurrence by 3.9% and 5.02%, respectively.

When we come to family level covariates, the risk of recurrence of dropout was significantly related with family income, parental educational, parental encouragement score, work demand score of parents and family shock score. High family income, parental education and parental encouragement score lower the risk of recurrence of school dropout by 2.16%, 3.2% and 4.39%, respectively, whereas high parental work demand score and family shock score increase the recurrence of dropout, respectively, by 4.14% and 4.02%.

The results also showed that the covariates considered at school level are significantly contributing to the recurrence of dropout. Primary school students in rural localities are at a higher risk of experiencing recurrent dropout compared to those attending schools in urban areas. Moreover, as the distance to school increases by one hour, the risk of recurrence of dropout increases by 31.5% (HR = 1.315; 95% CI: 1.096 – 1.579). Large class size and school size also increase the risk of recurrence of dropout by 0.8% and 10.1%, respectively.

In summary, late school enrollment, higher birth order in the family, and frequent absenteeism from student level; high work demand score of parents and family shock score from family level; and long distance, large class and school sizes from school level, increase the recurrence of dropout. Factors that were found to reduce the risk of recurrence of dropout from school were: spending more time on study, high family income, parental education, and high student motivation, parental encouragement, and teacher & school characteristic scores.

5. Conclusions
In this study, 1360 primary school pupils were followed up for seven years and their school dropout history during the study period was analyzed. The students experienced a maximum of four recurrences of school dropouts. Since events from the same student are expected to be correlated, the ordinary Cox proportional model is not appropriate to handle such dependence of events. An appropriate choice would rather be the generalized Cox regression model which can handle the dependence of events.

Our results clearly identified the main causal risk factors for the recurrence of school dropout. Student discipline (absenteeism and time devoted to study), student motivation about education and living condition were found to be important risk factors. Parental education, parents’ demand for child labour
and family shock were some of family-related factors that are significantly related with recurrence of school dropout. Finally, the educational system and school & teacher related characteristics were important risk factors that are the responsibility of school authorities. Interventions targeting the most important risk factors identified in the study are recommended to alleviate the problem of recurrent school dropout of students.

### Table 4: Results of the fitted PWP model

| Covariate                        | Estimate | se(coef) | se(coef) Robust | Pr (>|z|) | HR | 95% CI of HR |
|----------------------------------|----------|----------|----------------|--------|----|---------------|
|                                  |          |          |                |        |    | LL           | UL          |
| **Student level Covariates**     |          |          |                |        |    |               |             |
| Entry age                        | 0.215    | 0.157    | 0.108          | 0.047* | 1.239 | 1.002 - 1.533 |
| Birth order                      | 0.036    | 0.015    | 0.013          | 0.006** | 1.037 | 1.010 - 1.064 |
| Time to study                    | -0.171   | 0.025    | 0.023          | 0.000** | 0.843 | 0.804 - 0.882 |
| Student absenteeism              | -0.493   | 0.120    | 0.113          | 0.000** | 0.610 | 0.488 - 0.763 |
| Peer pressure to dropout         | -0.559   | 0.184    | 0.166          | 0.000** | 0.571 | 0.412 - 0.792 |
| Student ethics                   | -1.473   | 0.264    | 0.228          | 0.000** | 0.229 | 0.146 - 0.358 |
| Student motivation score         | -0.708   | 0.147    | 0.140          | 0.000** | 0.492 | 0.372 - 0.650 |
| Living condition score           | -0.695   | 0.197    | 0.171          | 0.000** | 0.498 | 0.356 - 0.698 |
| **Family level covariates**      |          |          |                |        |    |               |             |
| Family income                    | -0.242   | 0.120    | 0.109          | 0.026* | 0.784 | 0.633 - 0.972 |
| Parental education               | -0.385   | 0.123    | 0.111          | 0.000** | 0.680 | 0.546 - 0.846 |
| Parental encouragement score     | -0.576   | 0.198    | 0.184          | 0.001** | 0.561 | 0.391 - 0.806 |
| Parental work demand score       | -0.533   | 0.134    | 0.124          | 0.000** | 0.586 | 0.459 - 0.748 |
| Family shock score               | -0.512   | 0.113    | 0.100          | 0.000** | 0.598 | 0.491 - 0.729 |
| **School level Covariates**      |          |          |                |        |    |               |             |
| School location (urban)          | -0.134   | 0.076    | 0.066          | 0.045* | 0.874 | 0.767 - 0.997 |
| School size                      | 0.097    | 0.045    | 0.040          | 0.017* | 1.101 | 1.017 - 1.193 |
| Class size                       | 0.008    | 0.004    | 0.003          | 0.009** | 1.008 | 1.002 - 1.015 |
| School distance from home        | 0.274    | 0.109    | 0.093          | 0.003** | 1.315 | 1.096 - 1.579 |
| Teacher characteristics score    | -0.535   | 0.254    | 0.235          | 0.022* | 0.585 | 0.369 - 0.928 |
| School characteristics score     | -0.757   | 0.209    | 0.183          | 0.000** | 0.468 | 0.327 - 0.671 |
| Education system score           | -2.537   | 0.419    | 0.382          | 0.000** | 0.079 | 0.037 - 0.167 |
| **Global Model Test**            |          |          |                |        |    |               |             |
| Likelihood ratio test            | 710.6    | 20       |                | 0.000   |      |               |             |
| Wald test                        | 896.2    | 20       |                | 0.000   |      |               |             |
| Score (log-rank) test            | 698.2    | 20       |                | 0.000   |      |               |             |

*Significant at the 5% level, **Significant at the 1% level
References


Bayesian Modeling of Incidence of Pregnancy among Women under ART Follow-up at Adare Hospital, Hawassa, Ethiopia

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Abstract

Background: HIV/AIDS is the most serious disease human kind has ever faced and a public problem, particularly, for women of childbearing age. For HIV infected women, the prospects of getting pregnant and giving birth to a healthy (HIV-free) baby could be significantly improved with increased access to antiretroviral therapy (ART). Despite this fact, HIV infected women largely shun pregnancy in fear of mother to child transmission of HIV.

Objective: The objective of this study was to investigate the likelihood of pregnancy among HIV/AIDS patient women under ART follow-up.

Methods: A retrospective cohort study was conducted based on secondary data obtained from the medical chart of HIV/AIDS patient women aged 15-49 years under ART follow-up from April 2008 to February 2015. Out of all women under ART follow-up in Adare Hospital, Hawassa, a sample size of 328 was selected by using simple random sampling. Bayesian binary logistic regression analysis was used to identify the significant factors of likelihood of pregnancy.

Results and conclusions: The results of this study revealed that 21.3% of women got pregnant during the follow-up period. Bayesian logistic regression analysis indicated that younger age, lower levels of education and advanced WHO clinical stages were associated with decreased likelihood of pregnancy among women under ART follow-up. On the other hand, longer time on ART and higher CD4 cell counts were positively related with the incidence of pregnancy. The predictors identified in this study could be used to care for those HIV/AIDS patient women who want to bear children.

Keywords: HIV/AIDS, antiretroviral therapy, Bayesian logistic regression, likelihood of pregnancy

1. Background

HIV/AIDS is one of the most critical diseases human kind has ever faced as well as a social dilemma. It has become one of the world’s most serious health and development challenges as well as a social
problem particularly among women of childbearing age (Cooper et al., 2007). HIV/AIDS compromises their immunity which further aggravates their chances of conception and supporting pregnancy to term. AIDS epidemic has now spanned more than three decades and was first recognized by the United States Center for Disease Control and Prevention in 1981. An estimated 35.3 million people were living with HIV worldwide in 2012. The number of new infections has declined by 33% from 2001 (3.4 million) to 2012 (2.3 million). Due to improved access to antiretroviral therapy (ART), the number of AIDS deaths has declined from 2.3 million in 2005 to 1.6 million in 2012 (UNAIDS, 2013).

The massive global expansion of access to antiretroviral treatment (ART) has transformed not only the HIV epidemic but the entire public health landscape, demonstrating that the right to health can be realized even in the most trying of circumstances. In low and middle income countries, about 1.6 million more people were receiving ART at the end of 2012 compared with a year earlier, with the greatest contribution coming from the WHO African Region. The 300000 people who were receiving ART in low and middle income countries in 2002 increased to 9.7 million in 2012. This figure represented 61% of all people who were eligible. The scaling up of ART averted an estimated 4.2 million deaths in these countries between 2002 and 2012. In the WHO African Region, which continues to bear the brunt of the HIV epidemic, more than 7.5 million people were receiving treatment at the end of 2012 compared to 50000 people a decade earlier (WHO, 2013).

In 2012, over 900000 pregnant women living with HIV received treatment for prevention of mother-to-child transmission (PMTCT) – one third more than in 2009. Expanding programs for PMTCT and the use of more effective ARV regimens helped prevent more than 800000 children from becoming newly infected between 2005 and the end of 2012. In the 21 African priority countries in the Global Plan, which account for about 90% of all pregnant women living with HIV and new infections among children globally, mother-to-child transmission rates declined overall from an estimated 26% [24-30%] in 2009 to 17% [15-20%] in 2012 due to increased access to ART (WHO, 2013).

The global roll-out of ART has contributed to a greater awareness of issues related to fertility and childbearing among HIV-infected women and men, particularly in sub-Saharan Africa where a large proportion of HIV-infected individuals are women in their reproductive years and the prevention of mother-to-child transmission of HIV is an ongoing challenge (WHO, 2008). Few qualitative studies from Africa suggest that HIV might modify but does not eliminate broader desires to have children and that ART use may be associated with increased fertility desires among HIV-infected women, possibly through increased hopes and planning for the future (Maier, 2008).
Based on EHNRI (2012) and EDHS (2011) estimates, about 760,000 people were living with HIV in Ethiopia in 2012. The country has recorded some modest progress with respect to access to ART for its population. For instance, the ART coverage has increased from 55% in 2011 to 60% in 2012. In the country, approximately 38,000 pregnant women were living with HIV and 15,828 (about 41%) of these have received a full course of effective ARV treatment to prevent mother to child transmission in 2012. This coverage was remarkably higher compared to a year earlier (24%).

A number of studies have been conducted in Sub-Saharan Africa focusing on the incidence of pregnancy among women under ART follow-up. However, as to the authors' knowledge, there were no studies on pregnancy status as well as pregnancy rate of women under ART follow-up in Ethiopia. This is particularly essential since there is broad access to ART with extremely large at-risk population of reproductive-age women in the country. To fill this gap, this study attempted to explore the likelihood of pregnancy and its potential predictors among HIV/AIDS patient women under ART follow-up at Adare Hospital, Hawassa, Ethiopia.

Therefore, the objective of this study is to investigate the likelihood of pregnancy among HIV/AIDS patient women under ART follow-up using Bayesian logistic regression analysis. The specific objectives of the study are:

1. To identify the major predictor variables of likelihood of pregnancy among women under ART follow-up.
2. To explore the pregnancy status of HIV/AIDS patient women under ART follow-up
3. To provide information to health workers, governmental & non-governmental organization and researchers.

2. Literature review

With the advent of antiretroviral therapy (ART) and heightened global support for HIV/AIDS treatment, HIV positive women are living healthier and longer lives (WHO, 2010). ART use is associated with significantly higher pregnancy rates among HIV-infected women in sub-Saharan Africa. While the possible behavioral or biomedical mechanisms that may underlie this association require further investigation, these data highlight the importance of pregnancy planning and management as a critical but neglected component of HIV care and treatment services. Such services are in a unique position to address the childbearing desires of HIV-infected individuals as well as to ensure safe pregnancy and delivery (Myer et al., 2010).
A study in Uganda by Makumbi et al. (2011) reported that the use of ART was associated with increased pregnancy rates in HIV positive women, while older age and use of family planning were associated with lower pregnancy prevalence. Moreover, the prevalence of pregnancy during ART use was higher among women with CD4 count of 100–250 compared to those with CD4<100, and women aged 35–45 years compared to those aged 15–24 years.

According to a study in urban Malawi (Tweya et al., 2013), the incidence of pregnancy was significantly and negatively associated with current age and WHO clinical stage at ART initiation. On the other hand, longer time on ART was associated with increased probability of becoming pregnant. A retrospective clinical cohort study in South Africa reported that rates of pregnancy were highest in women with CD4 cell counts in the range 350 – 500 and much higher in younger women compared to older women (Westreich et al., 2012).

A study by Myer et al. (2010) utilizing proportional hazards models revealed that the factors that were significantly associated with an increased risk of pregnancy of women under ART follow-up included younger age, increased duration of follow-up, lower levels of education, being married or cohabiting, nonuse of contraception, and higher current CD4 cell counts. On the other hand, more advanced HIV disease (as indicated by higher WHO staging) was strongly associated with reduced incidence of pregnancy.

According to a study by Kabami et al. (2014) in western Uganda, younger women, women who have fewer children and women who did not know their spouse’s HIV status were more likely to get pregnant. The study found no significant association between pregnancy and religion, WHO disease stage and CD4 cell count at enrollment. A study conducted in southeastern Brazil has shown that age, level of education, marital status, use of ART and CD4 cell count were associated with the risk of pregnancy. However, the number of living children and HIV-related conditions did not show a clear association with pregnancy (Ruth et al., 2010).

As discussed above, studies have been conducted in Sub-Saharan Africa focusing on the incidence of pregnancy among women under ART follow-up. However, as to the authors’ knowledge, there were no studies on pregnancy status as well as pregnancy rate of women under ART follow-up in Ethiopia. To fill this gap, this study attempted to explore the likelihood of pregnancy and its potential predictors among HIV/AIDS patient women in the country.
3. Methods

3.1 Study design, data source and sample size

A retrospective cohort study was conducted based on secondary data obtained from medical charts of HIV infected women from ART clinic of Adare Hospital, Hawassa, Ethiopia. All HIV/AIDS patient women aged between 15 and 49 years who were under ART follow-up for at least three months between April 2008 and February 2015 comprise the population under consideration. A random sample of 328 women under ART follow-up was selected using single population proportion formula with degree of precision 4%. The data were collected by data experts from the ART clinic of the hospital using a structured questionnaire prepared to explore the factors that are associated with likelihood of pregnancy. The questionnaire was pre-tested to assess its clarity, flow and consistency.

3.2 Variables in the study

a) Dependent variable
The dependent (outcome) variable of the study was pregnancy status of HIV/AIDS patient women under ART follow-up. For logistic regression analysis, those women who were pregnant at the time of the survey were coded as 1 and those who were not as 0.

b) Explanatory variables
Based on the literature reviewed, the independent variables that are used to explore the likelihood of pregnancy of HIV/AIDS patient women under ART follow up were age, level of education, occupation, marital status, religion, place of residence, number of children alive before ART follow-up, contraception use, WHO clinical stage, illness due to co-infection, body weight, time under ART follow-up, CD4 cell count and spouse's HIV status.

3.3 Logistic regression model

Logistic regression analysis extends the techniques of multiple regression analysis to research situations in which the dependent variable is categorical. Logistic regression allows one to predict a discrete outcome, such as group membership, from a set of independent variables that may be continuous, discrete, dichotomous, or a mix of any of these. It is much more relaxed and flexible in its assumptions than multiple regression analysis. Unlike multiple linear regression analysis, for instance, logistic regression does not have the requirements of the dependent variable to be normally distributed, linearly related, nor equal variance within each group (Hosmer & Lemeshow, 2000). Logistic regression has a peculiar property of easiness to estimate logit differences for data collected both retrospectively and
prospectively (Mc Cullagh and Nelder, 1983) and this has contributed a lot to its importance in application areas.

Binary logistic regression is a type of logistic regression that is used when the dependent variable is dichotomous and the predictor variables are of any type. It estimates the probability that a certain characteristic is present (in our case, the probability that a woman under ART follow-up is pregnant) given the values of explanatory variables. Suppose \( X_i = (x_{i1}, x_{i2}, \ldots, x_{ik})' \) denotes the vector of predictor variables for the \( i^{th} \) individual, \( i = 1, 2, \ldots, n \). The probability of success of the \( i^{th} \) individual (that is, the probability that the \( i^{th} \) woman under ART follow-up is pregnant) given her background characteristics \( X_i \) is given by:

\[
P_i = \text{Prob}(Y_i = 1| X_i) = \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \quad \text{........................................... (1)}
\]

where \( \beta = (\beta_1, \beta_2, \ldots, \beta_k)' \) is a vector of unknown parameters. The logit transformation of \( P_i \) is a linear function of the explanatory variables:

\[
\logit(P_i) = \ln \left[ \frac{P_i}{1 - P_i} \right] = X_i'\beta = x_{i1}\beta_1 + x_{i2}\beta_2 + \ldots + x_{ik}\beta_k \quad \text{.............................. (2)}
\]

This transformation has certainly helped the popularity of the logit model.

3.4 Bayesian logistic regression

Bayesian analysis is a statistical procedure that answers research questions by expressing uncertainty about unknown parameters using probabilities. It is based on the fundamental assumption that not only the outcome of interest but also all the unknown parameters in a statistical model are essentially random and are subject to prior beliefs.

Bayesian analysis starts with the specification of a posterior model. The posterior model describes the probability distribution of all model parameters conditional on the observed data and some prior knowledge. The posterior distribution has two components—a likelihood, which includes information about model parameters based on the observed data, and a prior, which includes prior information (before observing the data) about model parameters. The likelihood and prior models are combined using the Bayes rule to produce the posterior distribution: Posterior \( \propto \) Likelihood x Prior
3.4.1 Likelihood function
Each response $Y_i$ may be treated as a single draw from a Bernoulli distribution with probability of success equal to $P_i$, $i = 1, 2, \ldots, n$. Since the responses $Y_1, Y_2, \ldots, Y_n$ are assumed to be independent, their joint density is simply the product of Bernoulli probabilities. Thus, using equation (1), the likelihood function is given by:

$$L(\beta \mid Y_1, Y_2, \ldots, Y_n; X) = \prod_{i=1}^{n} P_i^{Y_i}(1-P_i)^{1-Y_i}$$

$$= \prod_{i=1}^{n} \left( \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \right)^{Y_i} \left( 1 - \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \right)^{1-Y_i} \right) \quad \text{…………………… (3)}$$

3.4.2 Prior distribution
To progress with the Bayesian analysis, it is necessary to specify a joint prior distribution over the parameter space. If no prior information on the model parameters exists or it is difficult to elicit or formalize, then initial uncertainty about the parameters can be quantified with a non-informative prior distribution. This is equivalent to utilizing just the information provided by the data in the analysis. Souza and Migon (2004) and Migon and Tachibana (1997) proposed independent normal priors for the components of $\beta$ with extremely small precisions. These priors are equivalent to non-informative priors on these parameters, with all parameter values treated as equally plausible, and are given by:

$$f(\beta_j) = \frac{1}{\sqrt{2\pi \sigma_j^2}} \exp \left\{ -\frac{1}{2} \left( \frac{\beta_j - \mu_j}{\sigma_j} \right)^2 \right\} \quad j = 1, 2, \ldots, k \quad \text{…………………… (4)}$$

The most common choice for $\mu_j$ is zero, and $\sigma_j$ is usually chosen to be large enough to be considered as non-informative, common choices being in the range from $\sigma_j = 10$ to $\sigma_j = 100$ (Rashwan et al., 2012).

3.4.3 Posterior distribution
The posterior distribution is obtained by multiplying the prior distribution over all parameters by the full likelihood function, and is given by:

$$p(\beta \mid Y; X) = \prod_{i=1}^{n} \left[ \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \right]^{Y_i} \left( 1 - \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \right)^{1-Y_i} \right] \times \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi \sigma_j^2}} \exp \left\{ -\frac{1}{2} \left( \frac{\beta_j - \mu_j}{\sigma_j} \right)^2 \right\} \quad \text{…………………… (5)}$$
3.4.4 Markov chain Monte Carlo
Posterior distributions are rarely available in analytical forms and often involve multidimensional integrals. They are commonly estimated via simulation. Markov chain Monte Carlo (MCMC) sampling is often used to simulate potentially very complex high-dimensional posterior distributions. MCMC is a simulation-based method of estimating posterior distributions. It produces a sequence or a chain of simulated values (MCMC estimates) of model parameters from the estimated posterior distribution. If the chain "converges", the sequence represents a sample from the desired posterior distribution. There are different MCMC methods to estimate the chains of simulated values. Two more commonly used MCMC methods are Metropolis-Hastings (MH) algorithm and Gibbs algorithm.

3.4.5 The Gibbs sampling algorithm
Gibbs sampling (Gibbs sampler) introduced by Geman and Geman (1984) is an MCMC algorithm for obtaining a sequence of observations which are approximated from a specified multivariate probability distribution when direct sampling is difficult. This sequence can be used to approximate the joint distribution of a set of random variables or the marginal distribution of one of the variables or some subset of the variables. As with other MCMC algorithms, Gibbs sampling generates a Markov chain of samples, each of which is correlated with nearby samples. As a result, care must be taken if independent samples are desired. Generally, samples from the beginning of the chain (the burn-in period) may not accurately represent the desired distribution and are usually discarded. For this reason, MCMC algorithms are typically run for a large number of iterations in the hope that convergence to the target posterior will be achieved (Gelfand and Smith, 1990). In this study, the Gibbs sampler algorithm was used to estimate the marginal posterior distribution for each of the parameters by WinBUGS software.

3.4.6 Assessment of convergence of MCMC algorithm
The basic idea on an MCMC algorithm is to create a Markov process that has a stationary distribution which is the same as the posterior distribution of interest. Thus, technically speaking, convergence occurs when the generated Markov chain converges in distribution to the posterior distribution of interest. In an attempt to perform some kind of statistical analysis to assess the convergence of MCMC algorithms, a number of convergence diagnostics have been suggested (e.g., Brooks and Gelman, 1998; Cowles & Carlin, 1996; Brooks & Roberts, 1998). These include trace (time series) plots, autocorrelation plots, Gelman-Rubin statistics and density plots. For instance, a trace plot illustrates the values of the simulated parameters against the iteration number and connects consecutive values with a line. For a well-mixing parameter, the range of the parameter is traversed rapidly by the MCMC chain, which makes the drawn lines look almost vertical and dense. Sparseness and trends in the trace plot of a parameter suggest
convergence problems. For a given parameter, the Gelman-Rubin statistic compares the within-chain and between-chain variabilities. The model is judged to have converged if the ratio of between to within variability is close to one.

Once we confirm that convergence has been achieved, further iterations are needed to obtain samples for posterior inference. To assess the accuracy of the posterior estimates, we can use the Monte Carlo error for each of the parameters. As a rule of thumb, if the MC error is less than 5% of its posterior standard deviation, then we can conclude that the posterior density is estimated with accuracy.

4. Results

The objective of the study was to identify factors that affect the likelihood of pregnancy of HIV/AIDS patient women under ART follow-up. Among the 328 women included in the analysis, 21.3% had pregnancy during the study period. The Chi-square test of association was employed to examine the association between the response variable (whether a woman under ART follow-up is pregnant or not) and predictor variables. The results revealed that pregnancy of women under ART follow-up is significantly associated with WHO clinical stage, spouse's HIV status, marital status, educational level, occupation, contraceptive use, number of children before ART follow-up, age, CD4 cell count and time under ART follow-up. The percentage distribution reveals that the likelihood of pregnancy increases with the level of education and CD4 cell counts of women under ART follow-up, and decreases with increases in age and WHO clinical stages.

Bayesian analysis was utilized to estimate the parameters of the binary logistic regression model. The Gibbs sampler algorithm was implemented with 20000 iterations in three different chains, and 5000 burn-in terms were discarded from each so as to get 45000 samples from the posterior distribution. Table 1 presents the Bayesian logistic regression results based on the sample obtained from the joint posterior distribution.

The results indicate that WHO clinical stage, marital status, spouse's HIV status, educational level, contraception use, number of children before ART follow-up, occupation, CD4 cell count, time under ART follow-up and age were significant predictor variables of likelihood of pregnancy among women under ART follow-up (since the 95% credible intervals do not contain zero for at least one category of predictor variables). From the posterior means we can see that the likelihood of pregnancy was positively associated with time under ART follow-up, educational level and CD4 cell count. In contrast, WHO clinical stage, age and number of children alive at ART initiation were negatively associated with likelihood of pregnancy.
Table 1. Summary statistics of the posterior distribution of the model parameters

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Category</th>
<th>Mean ((\hat{\beta}))</th>
<th>S.E.</th>
<th>MC Error</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td>-3.383</td>
<td>0.181</td>
<td>0.102</td>
<td>-10.230</td>
<td>2.693</td>
</tr>
<tr>
<td>Time under ART follow-up</td>
<td>25-48 month</td>
<td>4.542*</td>
<td>0.067</td>
<td>0.012</td>
<td>2.288</td>
<td>7.080</td>
</tr>
<tr>
<td></td>
<td>&gt; 48 months</td>
<td>3.935*</td>
<td>0.072</td>
<td>0.013</td>
<td>1.455</td>
<td>6.565</td>
</tr>
<tr>
<td></td>
<td>(\leq) 24 months (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WHO clinical stage</td>
<td>Stage II</td>
<td>-2.118*</td>
<td>0.067</td>
<td>0.013</td>
<td>-4.204</td>
<td>-0.151</td>
</tr>
<tr>
<td></td>
<td>Stage III</td>
<td>-7.611*</td>
<td>0.103</td>
<td>0.023</td>
<td>-11.490</td>
<td>-4.180</td>
</tr>
<tr>
<td></td>
<td>Stage IV</td>
<td>-6.779*</td>
<td>0.142</td>
<td>0.021</td>
<td>-12.120</td>
<td>-1.965</td>
</tr>
<tr>
<td></td>
<td>Stage I (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spouse HIV status</td>
<td>Positive</td>
<td>2.644*</td>
<td>0.062</td>
<td>0.013</td>
<td>0.490</td>
<td>4.912</td>
</tr>
<tr>
<td></td>
<td>Unknown</td>
<td>-1.072</td>
<td>0.060</td>
<td>0.008</td>
<td>-3.226</td>
<td>1.038</td>
</tr>
<tr>
<td></td>
<td>Negative (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marital status</td>
<td>Unmarried</td>
<td>-6.626*</td>
<td>0.091</td>
<td>0.015</td>
<td>-10.060</td>
<td>-3.595</td>
</tr>
<tr>
<td></td>
<td>Divorced</td>
<td>-7.185*</td>
<td>0.091</td>
<td>0.016</td>
<td>-10.640</td>
<td>-4.199</td>
</tr>
<tr>
<td></td>
<td>Widowed</td>
<td>-6.702*</td>
<td>0.106</td>
<td>0.017</td>
<td>-10.770</td>
<td>-3.237</td>
</tr>
<tr>
<td></td>
<td>Married (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Educational level</td>
<td>Primary</td>
<td>1.515</td>
<td>0.071</td>
<td>0.019</td>
<td>-0.936</td>
<td>4.104</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>1.438</td>
<td>0.081</td>
<td>0.028</td>
<td>-1.365</td>
<td>4.417</td>
</tr>
<tr>
<td></td>
<td>College and above</td>
<td>2.913*</td>
<td>0.083</td>
<td>0.024</td>
<td>0.048</td>
<td>5.944</td>
</tr>
<tr>
<td></td>
<td>No education (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contraception uses</td>
<td>Rarely</td>
<td>-1.708</td>
<td>0.081</td>
<td>0.009</td>
<td>-4.604</td>
<td>1.131</td>
</tr>
<tr>
<td></td>
<td>Mostly</td>
<td>-5.610*</td>
<td>0.082</td>
<td>0.015</td>
<td>-8.692</td>
<td>-2.904</td>
</tr>
<tr>
<td></td>
<td>Always</td>
<td>-5.463*</td>
<td>0.091</td>
<td>0.015</td>
<td>-9.009</td>
<td>-2.530</td>
</tr>
<tr>
<td></td>
<td>Never use (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of children alive</td>
<td>1-2 child</td>
<td>-3.834*</td>
<td>0.059</td>
<td>0.011</td>
<td>-6.051</td>
<td>-1.854</td>
</tr>
<tr>
<td></td>
<td>3 or more children</td>
<td>-2.728*</td>
<td>0.074</td>
<td>0.012</td>
<td>-5.445</td>
<td>-0.194</td>
</tr>
<tr>
<td></td>
<td>No child (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD4 Count</td>
<td>250-350</td>
<td>-4.547*</td>
<td>0.099</td>
<td>0.027</td>
<td>-8.150</td>
<td>-1.088</td>
</tr>
<tr>
<td></td>
<td>351-500</td>
<td>2.491</td>
<td>0.085</td>
<td>0.032</td>
<td>-0.389</td>
<td>5.693</td>
</tr>
<tr>
<td></td>
<td>&gt;500</td>
<td>3.699*</td>
<td>0.079</td>
<td>0.026</td>
<td>1.110</td>
<td>6.704</td>
</tr>
<tr>
<td></td>
<td>&lt;250 (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Occupation</td>
<td>Employed</td>
<td>4.018*</td>
<td>0.069</td>
<td>0.017</td>
<td>1.664</td>
<td>6.636</td>
</tr>
<tr>
<td></td>
<td>Housewife</td>
<td>0.936</td>
<td>0.065</td>
<td>0.011</td>
<td>-1.345</td>
<td>3.273</td>
</tr>
<tr>
<td></td>
<td>Student</td>
<td>-3.240</td>
<td>0.159</td>
<td>0.012</td>
<td>-9.124</td>
<td>2.232</td>
</tr>
<tr>
<td></td>
<td>Unemployed (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>25-29</td>
<td>0.633</td>
<td>0.065</td>
<td>0.015</td>
<td>-1.633</td>
<td>2.985</td>
</tr>
<tr>
<td></td>
<td>30-34</td>
<td>-3.661*</td>
<td>0.087</td>
<td>0.019</td>
<td>-6.909</td>
<td>-0.727</td>
</tr>
<tr>
<td></td>
<td>35-39</td>
<td>-6.398*</td>
<td>0.105</td>
<td>0.018</td>
<td>-10.330</td>
<td>-2.899</td>
</tr>
<tr>
<td></td>
<td>40-49</td>
<td>-4.836*</td>
<td>0.142</td>
<td>0.018</td>
<td>-10.190</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>15-24 (ref.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ref. = reference category, * = significant at the 5% level

4.1 Assessment of convergence and accuracy of the fitted model

The convergence of the parameter estimates was assessed using the trace, autocorrelation and density plots as well as Gelman-Rubin statistics.
The trace plots of all parameter estimates under consideration look like a horizontal band (the three independently generated chains appear to be overlapping on one another), with no long upward or downward trends. Thus, we are reasonably confident that convergence has been achieved for the estimated parameters (that is, the Markov chain has converged to its stationary distribution for each of the parameter estimates). The time series plots for time under ART follow-up (25-48 months) and WHO clinical stage II are shown in Figure 1 for illustration.

![Figure 1: Trace plots of selected regression coefficient estimates](image1)

The realized values during MCMC are not independent of each other. The autocorrelation plot is a popular measure of this dependence. High autocorrelations indicate slow mixing within a chain, and hence, slow convergence to the posterior distribution. In this study, we observe quick drop-off in the autocorrelation functions of all parameter estimates as the number of lags increases. This is an indication that the sampler explored the posterior distribution much quicker. For illustration, the autocorrelation plots of time under ART follow-up (25-48) and WHO clinical Stage II are displayed in Figure 2.

![Figure 2: Autocorrelation plots of selected regression coefficient estimates](image2)

The Gelman-Rubin statistics which compare the within-chain and between-chain variabilities are alternative measures of convergence. This ratio converged to approximately one for each of the parameter estimates considered in this study. Thus, we can safely conclude that convergence of chains has been achieved. The Gelman-Rubin statistics for time under ART (25-48 months) and WHO clinical stage II are shown below.
The density plots (marginal posterior distributions) for beta parameters obtained from the joint distribution were found to be symmetric, unimodal and approximately normally distributed. This is an indication that the Markov chain has attained its posterior distribution. Figure 4 displays the density plots for time under ART (25-48 months) and WHO clinical stage II.

The accuracy of posterior estimates can be assessed by comparing the Monte Carlo error for each of the parameters with the posterior standard deviation. In this study, the MC error for each of the significant parameters was less than 5% of its posterior standard deviation. This indicates that the posterior density is estimated with accuracy and that the fitted model is adequate for posterior inference.

4.2 Discussion

The duration of time under ART follow-up was found to be a significant predictor of likelihood of pregnancy. Women with ART follow-up period of greater than 24 months were more likely to become pregnant compared to those who were under ART follow-up for less than 24 months. This indicates that longer time on ART was associated with increased probability of becoming pregnant. This result is supported by the findings of Tweya et al. (2013) and Myer et al. (2010) in which the risk of pregnancy appeared to increase continuously with increasing duration of follow-up in women on ART.

The study has shown that WHO clinical stage was a significant predictor of likelihood of pregnancy among women under ART follow-up. Women in advanced WHO clinical stages were less likely to become pregnant as compared to those in stage I. This is result is similar with that reported by Tweya et al. (2013) and Myer et al. (2010). Our result also revealed that lower CD4 cell counts during follow-up were associated with lower incidence of pregnancy. For instance, women with CD4 cell counts less than 250
were less likely to be pregnant compared to those whose CD4 cell counts were in excess of 500. Various studies have reported similar findings (Westreich et al., 2012; Makumbi et al, 2011; Ruth et al., 2010; Myer et al., 2010).

The other significant predictor was level of education. The results revealed that educational level was positively associated with the likelihood of pregnancy. In particular, women under ART follow-up with college and above educational level were more likely to become pregnant compared to non-educated women. This result is consistent with those of Myer et al. (2010) who reported that lower levels of education are significantly associated with an increased risk of pregnancy of women under ART follow-up. Marital status was found to be significantly associated with the likelihood of pregnancy among AIDS patient women under ART follow up. Our result indicated that women who were married or cohabiting at enrollment were more likely to become pregnant. This finding is also supported by similarly studies (Makumbi et al, 2011; Kabami et al., 2014; Ruth et al., 2010). Contraception use was another significant predictor of likelihood of pregnancy. The likelihood of pregnancy was highest for women under ART follow-up who never used contraception, and lowest for women who used contraception (mostly or always). This result is consistent with the studies of Kabami et al. (2014) and Myer et al. (2010) in which women who reported any contraceptive use during follow-up had lower pregnancy rates than those who did not.

The results of this study indicated that the number of children alive before ART follow-up had a significant influence on likelihood of pregnancy, that is, women who had at least one child at ART initiation were less likely to be pregnant compared to those who had no children. The result is consistent with the findings of Kabami et al. (2014). Occupation of women was also a significant factor of likelihood of pregnancy. Employed women were more likely to become pregnant compared to unemployed women, whereas there was no significant difference in the likelihood of pregnancy among students and housewives compared to unemployed women. This result is inconsistent with that reported by Tweya et al. (2013) in Malawi.

Age was significantly and negatively related with the likelihood of pregnancy among women under ART follow-up. The likelihood of pregnancy was significantly higher for women in the age groups 15-24 and 25-29 compared to those in older age groups. In general, when age of women increased, the probability of becoming pregnant decreased. This finding is supported by similar studies (Westreich et al., 2012; Tweya et al. 2013; Makumbi et al, 2011; Kabami et al., 2014; Ruth et al., 2010).
5. Conclusions and recommendations

The main purpose of this study was to identify the factors that are associated with the likelihood of pregnancy among HIV/AIDS patient women of reproductive ages (15-49 years) under ART follow-up. From the descriptive analysis, the proportion of women under ART follow-up who became pregnant during the study period was 21.3%. In Bayesian analysis, the posterior inference was implemented by Gibbs sampler algorithm. MCMC diagnostic measures (trace, posterior density & autocorrelation plots and Gelman-Rubin statistics) suggested that each of the parameter estimates has converged to its posterior distribution.

The factors that were significantly associated with an increased risk of pregnancy included younger age, lower levels of education, being married, non-use of contraception, longer time on ART follow-up and higher CD4 cell counts. On the other hand, having live children before initiation of ART and advanced WHO clinical stages were negatively associated with the incidence of pregnancy of mothers under ART follow-up.

Recommendations

Based on the results of this study, the following recommendations are forwarded:

- Our results revealed that the likelihood of pregnancy among HIV/AIDS patient women has increased with an increase in duration of ART follow-up. This demands the inclusion of pregnancy planning and management strategy as a critical component of HIV care and treatment services.

- Women in advanced WHO clinical stage and low CD4 cell counts were found less likely to be pregnant. This might probably be attributed to their apprehensions towards their health and that of the child. Appropriate planning prior to pregnancy and administration of effective interventions during pregnancy will help to address the childbearing desires of HIV-infected women as well as to ensure safe pregnancy and delivery.

- Among the women under ART follow-up in this study, an overwhelming majority (64%) had never used contraceptives. Low prevalence of contraceptive use underscores the importance of addressing fertility-related issues within HIV care and treatment programs.

References


Statistical Analysis of the Volatility of the Export Price of Coffee in Ethiopia

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Abstract

Ethiopia is credited as being the birthplace of coffee. In Ethiopia, the export price of coffee is among the most volatile agricultural commodity prices. This study attempted to identify and analyze the factors that are correlated with the volatility of the export price of coffee in Ethiopia using data spanning from January 2002 to June 2016. Most of the series considered in this study were found to be integrated of order one or I(1). The monthly export price return series exhibited the stylized facts of financial time series such as volatility clustering and leptokurtosis. Thus, the ARCH family models (GARCH and EGARCH models) with ARMA mean equations were fitted to the data. ARMA (1, 1)-EGARCH (3, 1) model with normal error distributional assumption was selected as the best-fit model since the asymmetric term was significant and the forecasting error as well as AIC and BIC were smaller. Among the exogenous variables considered in this study, fuel oil price, non-food price, exchange rate and some of the seasonal dummies were found to have a statistically significant effect on the volatility of the export price return series. The significance of the asymmetric term indicates that an unanticipated decrease in the export price of coffee had a significantly higher impact on price volatility than unanticipated increase in the price. Additionally, past shocks and lagged volatility of export price had statistically significant effect on the price volatility of coffee.

Keywords: Export price of coffee, Stationarity, Volatility, ARMA, EGARCH

1. Introduction

Most of the literature regarding commodity markets has focused on price levels rather than price volatilities. There is a need for a clear distinction between these two aspects. Most internationally traded agricultural commodities are storable so that high price volatility is indeed more likely when prices are high and stocks are low. Nevertheless, a qualified discussion of the drivers of price volatility requires a careful distinction between drivers of price levels and drivers of price volatility.

In the past, international coffee prices have been quite volatile and such volatilities were driven by a mix of climatic conditions in the largest coffee-producing countries, expectations about future prices, changes
in demand and interest rates, as well as speculation (Deaton 1999). Price volatility in commodity markets has been studied extensively in the academic literature. According to Newbery (1989), commodity prices in general, and agricultural commodity prices in particular, are renowned for their continuously volatile nature.

Exchange rate can affect commodity prices through a number of channels, including international purchasing power. Using co-integration analysis, Gilbert (1989) found that domestic food price is significantly influenced by exchange rate in the long-run. According to Liefert and Persaud (2009), exchange rate movements can influence countries’ domestic and export prices, thereby affecting incentives to produce, consume and trade goods.

According to the finding of Frankel (1986) using linear regression model, interest rate has a significant influence on the agricultural market in general by affecting the cost of holding inventory, investment decision (land, machinery and input purchases) and the overall farm business risks. Frankel (2006) also suggested that interest rate plays an important role in determining total commodity supply and rising interest rate often causes the commodity price to fall. Moreover, the study reported that the level of real interest rate affects commodity prices through a number of supply and demand channels. Hammoudeh and Yuan (2008) using GARCH model also found that rising interest rates have a dulling effect on price volatility.

Swaray (2007) found that fluctuations in business cycles and macroeconomic variables, including fuel oil prices, have significant impact on non-fuel primary commodities. Baffes (2007) found that fuel oil price affects the price of agricultural commodities. Maurice & Davis (2011) using Granger Causality models found that there is a long-run causality between oil prices and coffee. Using EGARCH model, Maurice & Davis (2011) reported that a large increase in oil prices (listed as a negative shock) has a lower impact on coffee price variability than a steep decline in oil prices (positive shock) of a similar magnitude, that is, in a world of high oil prices, coffee price volatility is not as excessive as in a context of low oil prices.

Chambers (1984) studied the extent to which macroeconomic factors alter agricultural prices and found that real, aggregated agricultural prices have not been altered by the level of general inflation. According to Moledina et al. (2003), the production of a particular agricultural commodity is dependent on growing seasons. The dependence of production of a specific commodity on growing seasons can cause seasonal variations in supply and demand. The variation in supply and demand of commodities in turn causes seasonal fluctuations in prices.
Volatility in the price of coffee influences large proportion of the population all along the coffee commodity chain in Ethiopia (Yohannes, 2010). Worako et al. (2011) have made a distinction between producer, wholesale and export prices in order to compare the price risk faced by the respective participants in the coffee chain. The study also tried to compare the price volatility of Ethiopian and Brazilian coffee (major coffee producing countries in the world) and found that coffee prices within Ethiopia were more volatile than in Brazil. Moreover, producer prices were found to be the most volatile, followed by wholesale prices and export prices.

In the past, coffee prices have exhibited high inter-year (seasonal) variations in Ethiopia. These variations are a combined effect of the factors reflecting domestic supply and the periodic trends of the global coffee demand and supply situation (ECX, 2012). Coffee production and marketing in Ethiopia have influenced the export price of coffee due to low quality, poor market infrastructure, and long and traditional marketing channels. These could cause sudden fluctuations in export price of coffee, worsen price instability and create serious problems in the national income.

Price volatility does not usually affect a single market, but spillovers abound. Zerihun & Worako (2011) studied the interrelationships among producer, auction and world prices based on monthly price data ranging from October 1992 to September 2006 in Ethiopia. Using VAR and VECM models, they found that there is a unidirectional transmission of shocks from the world price to the auction price and then to the producer price. The study also revealed the presence of asymmetries in price transmissions and adjustments in the auction market, and weak interrelationship between producer and world prices causing producer prices to be less responsive to changes in the world price. In general, their results suggest that coffee growers benefit little from positive changes in the world price compared to participants in the auction markets.

Worako et al. (2008) studied the price of the producer, auction and Free-On-Board price (FOB) using data that extend from October 1992 to September 2006. The price data included in the analysis were comprised of four major Ethiopian coffee types by original growing region (Sidama, Harar, Wollega and Jimma). Using ECM, they found that there is a strong, long-run relationship among growers, wholesaler and exporter prices. The estimation of the ECM shows the transmission of price shocks from the world price to the auction price and then to the product price. According to Worako et al. (2008), the domestic price of coffee in Ethiopia adjusts more rapidly to world price changes.

Several studies related to the price of coffee have been conducted in Ethiopia (e.g., Bart et al., 2014; Worako et al., 2011). However, as to the authors’ knowledge, none of them have focused on identifying
significant drivers of the export price volatility of coffee. To fill this gap, this study attempted to investigate the potential factors that affect the volatility of the export price of coffee in Ethiopia by utilizing financial time series models.

2. Methodology

2.1 Variables considered in the study

The response variable in this study is the monthly export price return of coffee in Ethiopia. The explanatory variables that are assumed to affect the export price volatility of coffee in Ethiopia include the following:

- Exchange rate (US Dollar to Birr).
- Saving interest rate: the interest rate paid to deposit account holders for accounts like certificates of deposit and savings accounts
- Fuel oil price: the price of one metric ton fuel oil (in Birr).
- Inflation rate for food items: A quantitative measure of the rate at which the average price level of a basket of food items in an economy increases over a period of time
- The inflation rate for non-food items: A quantitative measure of the rate at which the average price level of non-food items in an economy increases over a period of time
- Domestic price: the price of Ethiopian coffee in the Ethiopian market.
- GDP: total output or the output values of goods and services at market prices excluding net income from abroad (in millions of Birr).
- Total government revenue: revenues earned by the government which are received from sources such as taxes levied on the incomes and wealth accumulation of individuals and corporations and on the goods and services produced, exports and imports, non-taxable sources such as government-owned corporations' incomes, central bank revenue and capital receipts in the form of external loans and debts from international financial institutions.

2.2 Statistical model specification

2.2.1 Mean model

The Autoregressive Moving Average (ARMA (p, q)) model is given by (Box and Jenkins, 1976):

\[ Y_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t \]  

\[ \epsilon_t \]
where $Y_t$ is the average monthly export price return of coffee at time $t$, $\varepsilon_t$ is a white noise random error term, and $p$ and $q$ are the autoregressive and moving average orders, respectively.

### 2.2.2 Variance Model

The Generalized Autoregressive Conditionally Heteroscedastic (GARCH $(r, m)$) model for the conditional variance of the residuals at time $t$ is given by (Bollerslev, 1986):

$$
\varepsilon_t = \sigma_t \psi_t
$$

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 \hspace{1cm} (2)
$$

where $\{\psi_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance one, $\sigma_t^2 = \mathbb{E}(\varepsilon_t^2 | \psi_{t-1})$, and $r$ and $m$ are the ARCH and GARCH orders, respectively. Here $\psi_{t-1}$ denotes the information set up to and including time $(t-1)$. The restrictions $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, 2, \ldots, r$ and $\beta_j \geq 0$ for $j = 1, 2, \ldots, m$ are imposed to ensure that the conditional variance is positive. The process is covariance stationary if and only if $\sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{m} \beta_j < 1$. The GARCH $(r, m)$ model with explanatory variables is given by:

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 + f(X_t, \gamma) \hspace{1cm} (3)
$$

Here the function $f(X_t, \gamma)$ is assumed to be strictly positive, where $X_t = (X_{1t}, X_{2t}, \ldots, X_{kt})'$ is a vector of explanatory variables at time $t$ and $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_k)'$ is a vector of regression coefficients that quantify the effect of explanatory variables on the conditional variance.

A variant of the GARCH model which allows for asymmetric effects is the Exponential GARCH (EGARCH) model introduced by Nelson (1991). The EGARCH $(r, m)$ model for the conditional variance of the residuals at time $t$ is given by:

$$
\varepsilon_t = \sigma_t \psi_t
$$

$$
\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{r} [\alpha_i \varepsilon_{t-i} + \delta_i \psi_{t-i}] + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-j}^2) \hspace{1cm} (4)
$$
Here $\delta_1, \delta_2, \ldots, \delta_r$ quantify the magnitude of asymmetric effects. No restriction is imposed on the coefficients of the model since the logarithmic transformation overcomes the positivity constraint. The presence of leverage effects can be tested by the hypothesis that $\delta_i \neq 0$, $i = 1, 2, \ldots, r$.

GARCH family model building process takes into account the following tests:

a) **Test for the presence of a unit root**
Unit root tests need to be performed to examine the stationarity of the series under study. In this regard, the Augmented Dickey Fuller (ADF) test (Dickey and Fuller, 1979) and Phillips-Perron (PP) test (Phillips and Perron, 1988) have been used for the said purpose.

b) **Test for ARCH effects**
In financial time series, ARCH effect is common (Asteriou and Hall, 2007). In this study, the Lagrange Multiplier (LM) test was used to check for the presence of ARCH effects by testing the significance of serial correlations in the squared residuals for the first few lags.

c) **Test of normality**
Financial time series often have thick tailed distribution indicating a departure from the normal distribution. Thus, the Jarque-Bera test was used to test the normality of the time series under consideration.

2.2.3 **Order selection for GARCH family models**
Once we are certain that ARCH effects are present in the financial time series under study, the next step involves identifying the appropriate orders for GARCH family models using Akaike Information Criterion (AIC) and Schwarz Bayesian Information Criterion (SBIC).

2.2.4 **Parameter estimation for GARCH family models**
Since GARCH family models are no longer of the usual linear form, OLS cannot be used for model estimation. One of the reasons is that OLS minimizes the residual sum of squares which depends only on the parameters of the conditional mean equation, and not on the parameters of the conditional variance equation. Another reason is that, under the presence of ARCH effects, OLS estimation is not efficient since the conditional variance of the errors is not constant. Due to these reasons, the maximum-likelihood (ML) estimation method was used to estimate the parameters of GARCH family models by assuming normal, student-$t$ and Generalized Error Distribution (GED) for the error term.
The log-likelihood functions under different distributional assumptions are:

a) If $\varepsilon_i \sim N(0, \sigma_i^2)$, the log-likelihood function is given by:

$$
\ln(L) = \sum_{t=1}^{T} \left(-\frac{1}{2} \log(2\pi) - \frac{1}{2} \ln(\sigma_i^2) - \frac{\varepsilon_i^2}{2\sigma_i^2}\right) 
$$

$$
\text{.................................(5)}
$$

b) If $\varepsilon_i \sim t(0, \sigma_i^2, \nu)$, where $\nu$ is the number of degrees of freedom, then the log-likelihood function is given by:

$$
\ln(L) = \sum_{t=1}^{T} \left[\log\left(\frac{\nu+1}{2}\right) - \frac{1}{2} \ln\left(\frac{\pi(\nu-2)}{\sigma_i^2}\right) - \left(\frac{\nu+1}{2}\right) \ln\left(1 + \frac{\varepsilon_i^2}{\sigma_i^2(\nu-2)}\right)\right] 
$$

$$
\text{.................................(6)}
$$

where $\Gamma(*)$ is the Gamma function.

c) Under the assumption that the errors follow independent GED with mean zero, variance $\sigma_i^2 > 0$ and degree of freedom (shape parameter) $\nu > 0$, that is, $\varepsilon_i \sim GED(0, \sigma_i^2, \nu)$, we have:

$$
\ln(L) = \sum_{t=1}^{T} \left[\ln\left(\frac{\nu}{\lambda}\right) - \ln\left(\frac{1}{\nu}\right) - \frac{1}{2} \ln(\sigma_i^2) - \left(1 + \frac{1}{\nu}\right) \ln(2) - \frac{1}{2} \left(\frac{\varepsilon_i^2}{\lambda^2 \sigma_i^2}\right)^{\nu/2}\right] 
$$

$$
\text{.................................(7)}
$$

The appropriate distribution for the residuals in the mean equation can be determined based on the forecasting ability of the models under the specified error distribution. The Root Mean Square Error (RMSE) and Theil Inequality Coefficient (U) are among the measures of the forecasting accuracy of ARCH-GARCH models.

### 2.2.5 Model adequacy checking

After fitting GARCH family models, it is necessary to check that the model actually does provide an adequate description of the time series under consideration. If the final model is good-fit to the data, then the partial autocorrelation function (PACF) of the squared standardized residuals should be indicative of a white noise process. The Ljung-Box test could be used in this regard. Moreover, the standardized residuals should be independent and identically distributed as standard normal even if the Student-t or the GED are assumed (Tsay, 2005). This can be checked using the Jarque-Bera test.
3. Result and discussion

3.1 Descriptive analysis

The data set used in this study consist of the monthly export price of coffee (in Birr per Kg), the domestic price of coffee (in Birr per Kg), food inflation, nonfood inflation, fuel oil price (in Birr per metric ton), real GDP (in millions of Birr), exchange rate (US dollar to Birr), saving interest rate and total government revenue (in millions of Birr) in Ethiopia from January 2002 to December 2016. Summary statistics of the export price of coffee and selected macro-economic variables are presented in Table 1 below.

The average monthly export price of coffee was Birr 44.494 ($1.58) per kg with a standard deviation Birr 28.329 ($1.01). The coefficient of variation of about 64% tells us that there is large variation in the monthly export price of coffee. Moreover, the coefficient of kurtosis is indicative of excess kurtosis (leptokurtic distribution). These features are also shared by most of the other covariates. The presence of a considerable variation within independent variables across time might induce volatility in the export price of coffee.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Coe. of Var.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export price of coffee</td>
<td>44.49</td>
<td>32.46</td>
<td>28.33</td>
<td>63.67</td>
<td>2.59</td>
<td>16.64</td>
</tr>
<tr>
<td>Domestic price of coffee</td>
<td>45.53</td>
<td>37.10</td>
<td>28.59</td>
<td>62.80</td>
<td>4.79</td>
<td>46.08</td>
</tr>
<tr>
<td>Food inflation</td>
<td>15.35</td>
<td>11.85</td>
<td>15.60</td>
<td>101.61</td>
<td>1.13</td>
<td>3.97</td>
</tr>
<tr>
<td>Non-food inflation</td>
<td>11.22</td>
<td>0.44</td>
<td>7.62</td>
<td>67.93</td>
<td>0.21</td>
<td>1.92</td>
</tr>
<tr>
<td>Fuel oil price</td>
<td>47588.98</td>
<td>7337.09</td>
<td>27887.30</td>
<td>58.60</td>
<td>3.43</td>
<td>14.59</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>13.08</td>
<td>10.34</td>
<td>4.81</td>
<td>36.73</td>
<td>0.44</td>
<td>1.48</td>
</tr>
<tr>
<td>Saving interest rate</td>
<td>4.11</td>
<td>4.25</td>
<td>0.92</td>
<td>22.48</td>
<td>-0.15</td>
<td>1.44</td>
</tr>
<tr>
<td>GDP</td>
<td>386086.50</td>
<td>274480.82</td>
<td>324329.02</td>
<td>84.00</td>
<td>0.63</td>
<td>1.83</td>
</tr>
<tr>
<td>Revenue</td>
<td>74261.66</td>
<td>47788.29</td>
<td>365667.03</td>
<td>492.40</td>
<td>0.97</td>
<td>2.75</td>
</tr>
</tbody>
</table>

Time series plot of the monthly export price of coffee (Birr per Kg) is shown in Figure 1. In general, the export price of coffee exhibited an increasing trend from 2002 up to 2016. Some fluctuations (ups and downs) are observed from 2009 to 2012 and around the end of the study period.
Figure 1: Time series plot of the monthly export price of coffee

The export price return series is plotted in Figure 2. We can observe that the return series revolves around a constant mean with no apparent trend. Moreover, the monthly export price return series exhibited the stylized facts of financial time series such as volatility clustering, that is, periods of high volatility followed by high volatility, and similarly for low volatility processes.

Figure 2: Time series plot of the monthly export price return of coffee

3.2 Tests of normality
Different literatures indicate that the distribution of price returns exhibit features such as leptokurtosis and volatility clustering (Cornew et al., 1984). In this study, the Jarque-Bera (JB) test has rejected the null hypothesis of normality for the monthly export price return series at the 1% level of significance. The non-normality of the return series might be due to the existence of excess kurtosis that we have observed from the descriptive statistics in Table 1.

3.3 Unit-root tests
The series should be stationary in order to fit a suitable time series model. The augmented Dickey-Fuller (ADF) test was used to check whether the series under consideration are stationary or not. It tests the null hypothesis that the series has a unit-root versus the alternative that the series is stationary. The ADF test
results are presented in Table 2. The results indicate that domestic price of coffee and food inflation are stationary at level, while all other macroeconomic variables are stationary at first difference.

Table 2: Unit-root test results

<table>
<thead>
<tr>
<th>Variables</th>
<th>At level</th>
<th></th>
<th></th>
<th>First difference</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF test statistic</td>
<td>p-value</td>
<td>ADF test statistic</td>
<td>p-value</td>
<td></td>
</tr>
<tr>
<td>Export price of coffee</td>
<td>-2.817</td>
<td>0.215</td>
<td></td>
<td>-20.426</td>
<td>0.000</td>
</tr>
<tr>
<td>Domestic price of coffee</td>
<td>-3.934</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food inflation</td>
<td>-5.192</td>
<td>0.009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-food inflation</td>
<td>11.218</td>
<td>0.245</td>
<td></td>
<td>-7.898</td>
<td>0.000</td>
</tr>
<tr>
<td>Fuel oil price</td>
<td>-2.879</td>
<td>0.245</td>
<td></td>
<td>-15.532</td>
<td>0.000</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>1.584</td>
<td>0.999</td>
<td></td>
<td>-11.299</td>
<td>0.000</td>
</tr>
<tr>
<td>Saving interest rate</td>
<td>-2.282</td>
<td>0.179</td>
<td></td>
<td>-13.273</td>
<td>0.000</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.056</td>
<td>0.951</td>
<td></td>
<td>-13.236</td>
<td>0.000</td>
</tr>
<tr>
<td>Revenue</td>
<td>2.554</td>
<td>1.000</td>
<td></td>
<td>-13.429</td>
<td>0.000</td>
</tr>
</tbody>
</table>

3.4 Mean equation specification and selection

In order to model the volatility of the return series, we need to specify their mean equation first. In the specification of the mean equation, the sample ACF and PACF plots of the stationary series can be used to tentatively identify the order of autoregressive terms and/or moving average terms. However, the final model is often selected based on information criteria.

In most applications, lower order ARMA models are often considered. Thus, the 15 combinations of AR (0 - 3) and MA (0 - 3) models were fitted in this study. Among these candidate models, ARMA (1,1) model was found to have the smallest AIC and SBIC, and hence, was used as the mean equation for the export price return of coffee. The fitted mean model is shown in Table 3 below.

Table 3: The fitted mean equation for average monthly export price return series

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. error</th>
<th>T-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.011</td>
<td>0.003</td>
<td>3.451</td>
<td>0.000</td>
</tr>
<tr>
<td>AR (1)</td>
<td>0.565</td>
<td>0.094</td>
<td>6.008</td>
<td>0.000</td>
</tr>
<tr>
<td>MA (1)</td>
<td>-0.898</td>
<td>0.051</td>
<td>-17.574</td>
<td>0.000</td>
</tr>
</tbody>
</table>

3.5 Tests for the presence of ARCH effects

ARCH effect is commonly found in financial time series (Cotter and Stevenson, 2006). Based on the residuals from the mean equation, it is possible to test for the existence of ARCH effects using the ARCH LM test. The results of the ARCH LM test for the residuals of the fitted ARMA (1,1) model for the export
price return series indicated that the current squared residual is significantly correlated with first two lags of squared residuals. Consequently, we need to use GARCH family models.

### 3.6 GARCH-family model selection

Even though ARCH models have attractive properties, a large number of lags (q), and thus, a large number of parameters, are required to obtain a good model fit. A GARCH model with low orders, on the other hand, results in a more parsimonious representation of the conditional variance process (Anderson, 2009). To choose the best-fit GARCH model among candidate volatility models, Akaike Information Criterion (AIC) and Schwarz Bayesian information Criterion (SBIC) are often used. Note that the AIC and BIC of the GARCH models are obtained by estimating the mean and variance equations simultaneously.

In our model selection procedure, various low order GARCH family models were considered. Among these models, ARMA (1,1)-EGARCH (3,1), ARMA (1,1)-EGARCH (2,3) and ARMA (1,1)-EGARCH (3,3) models with normal, Student’s-t and GED distributional assumptions for the residuals, respectively, were selected for the export price return volatility of coffee. To select the appropriate conditional volatility model among these candidate models, we considered their forecasting performance. The forecasting performance of the fitted GARCH family models was evaluated by RMSE and Theil inequality coefficients.

A summary of forecast accuracy measures for the candidate GARCH-family models is given in Table 4. We can observe that EGARCH (3,1) model with normal distributional assumption for the residuals possesses the smallest forecast accuracy measures, and that the asymmetric effect is significant. Thus, this model was selected to describe and analyze the export price volatility of coffee.

#### Table 4: Summary of forecast accuracy measures

<table>
<thead>
<tr>
<th>Model</th>
<th>Error distribution</th>
<th>Forecast accuracy measures</th>
<th>Asymmetric effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA (1,1)-EGARCH (3,1)</td>
<td>Normal</td>
<td>0.190</td>
<td>0.958</td>
</tr>
<tr>
<td>ARMA (1,1)-EGARCH (2,3)</td>
<td>GED</td>
<td>0.191</td>
<td>0.993</td>
</tr>
<tr>
<td>ARMA (1,1)-EGARCH (3,3)</td>
<td>Student-t</td>
<td>0.191</td>
<td>0.974</td>
</tr>
</tbody>
</table>

### 3.7 Parameter estimation

Once the ARMA (1,1)-EGARCH (3,1) model with normal distributional assumption for the residuals is selected, then the next step is to estimate the parameters of the model using the maximum likelihood approach. The results are given in Table 5.
Table 5: Fitted ARMA (1,1)-EGARCH (3,1) model under normal distributional assumption of the residuals for the return series of export price of coffee

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Std. Error</th>
<th>Statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.014</td>
<td>0.003</td>
<td>4.116</td>
<td>0.000*</td>
</tr>
<tr>
<td>AR (1)</td>
<td>-0.989</td>
<td>0.005</td>
<td>-210.171</td>
<td>0.000*</td>
</tr>
<tr>
<td>MA (1)</td>
<td>0.987</td>
<td>0.005</td>
<td>204.407</td>
<td>0.000*</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.852</td>
<td>0.679</td>
<td>-1.255</td>
<td>0.210</td>
</tr>
<tr>
<td>ARCH (-1)</td>
<td>0.958</td>
<td>0.182</td>
<td>5.272</td>
<td>0.000*</td>
</tr>
<tr>
<td>Asymmetric (1)</td>
<td>0.233</td>
<td>0.097</td>
<td>2.408</td>
<td>0.016**</td>
</tr>
<tr>
<td>EGARCH (-1)</td>
<td>1.376</td>
<td>0.082</td>
<td>16.878</td>
<td>0.000*</td>
</tr>
<tr>
<td>EGARCH (-2)</td>
<td>-1.235</td>
<td>0.064</td>
<td>-19.252</td>
<td>0.000*</td>
</tr>
<tr>
<td>EGARCH (-3)</td>
<td>0.638</td>
<td>0.077</td>
<td>8.242</td>
<td>0.000*</td>
</tr>
<tr>
<td>Domestic price of coffee</td>
<td>-0.014</td>
<td>0.026</td>
<td>0.013</td>
<td>0.068</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>0.401</td>
<td>0.132</td>
<td>3.027</td>
<td>0.003*</td>
</tr>
<tr>
<td>Food inflation</td>
<td>0.041</td>
<td>0.041</td>
<td>0.995</td>
<td>0.320</td>
</tr>
<tr>
<td>Fuel oil price</td>
<td>0.045</td>
<td>0.030</td>
<td>-1.243</td>
<td>0.044**</td>
</tr>
<tr>
<td>Non-food inflation</td>
<td>1.325</td>
<td>0.639</td>
<td>2.072</td>
<td>0.038**</td>
</tr>
<tr>
<td>Saving interest rate</td>
<td>-0.119</td>
<td>0.306</td>
<td>-0.388</td>
<td>0.699</td>
</tr>
<tr>
<td>Gov’t revenue</td>
<td>0.001</td>
<td>0.001</td>
<td>1.121</td>
<td>0.262</td>
</tr>
<tr>
<td>GDP</td>
<td>0.000</td>
<td>0.000</td>
<td>0.963</td>
<td>0.336</td>
</tr>
<tr>
<td>April</td>
<td>-3.223</td>
<td>0.933</td>
<td>-3.455</td>
<td>0.074</td>
</tr>
<tr>
<td>August</td>
<td>2.885</td>
<td>0.848</td>
<td>-3.402</td>
<td>0.001*</td>
</tr>
<tr>
<td>October</td>
<td>3.004</td>
<td>0.970</td>
<td>-3.096</td>
<td>0.002*</td>
</tr>
<tr>
<td>November</td>
<td>-4.014</td>
<td>0.641</td>
<td>-6.260</td>
<td>0.000*</td>
</tr>
<tr>
<td>December</td>
<td>-2.727</td>
<td>0.769</td>
<td>-3.547</td>
<td>0.009</td>
</tr>
<tr>
<td>March</td>
<td>-2.172</td>
<td>0.730</td>
<td>-2.975</td>
<td>0.283</td>
</tr>
<tr>
<td>February</td>
<td>0.467</td>
<td>1.046</td>
<td>-0.447</td>
<td>0.066</td>
</tr>
<tr>
<td>May</td>
<td>3.859</td>
<td>0.674</td>
<td>-5.726</td>
<td>0.000*</td>
</tr>
<tr>
<td>July</td>
<td>0.195</td>
<td>0.864</td>
<td>0.225</td>
<td>0.082</td>
</tr>
<tr>
<td>June</td>
<td>4.075</td>
<td>0.768</td>
<td>-5.306</td>
<td>0.000*</td>
</tr>
<tr>
<td>September</td>
<td>1.103</td>
<td>0.701</td>
<td>-1.573</td>
<td>0.012**</td>
</tr>
</tbody>
</table>

* and ** indicate significance at 1% and 5% level, respectively

3.8 Model diagnostics

The presence of remaining ARCH effects in the ARMA (1,1)-EGARCH (3,1) model was tested using the ARCH LM test. The test statistic was found to be insignificant, and hence, we do not have enough evidence to reject the null hypothesis that there is no ARCH left in the residuals. The Jarque-Bera test was used to test the normality of the residuals in the fitted EGARCH model. The result indicated that the null hypothesis of normality of the residuals cannot be rejected. Moreover, the skewness and kurtosis
coefficients of the standardized residuals from the fitted model were 0.258 and 2.687, respectively, indicating that the skewness and excess kurtosis have been considerably reduced.

3.9 Discussion of results

Among the explanatory variables which were considered in this study, domestic price of coffee, exchange rate, fuel oil price, non-food inflation rate and some of the seasonal dummies were found to be significant. On the other hand, food inflation rate, saving interest rate, total government revenue and GDP were found to have no significant influence on the current month export price volatility of coffee. The results also revealed significant effects of lagged innovations and lagged volatilities on the current conditional return volatility (see Table 5).

The coefficients of non-food inflation rate and exchange rate are positive and statistically significant. This indicates that increases in non-food inflation rate and exchange rate lead to an increase in the monthly export price volatility of coffee.

The coefficient of fuel oil price is also positive and statistically significant at the 5% level. The implication is that an increase in fuel oil price leads to an increase in the monthly export price volatility of coffee. Coffee production is increasingly mechanized and uses various chemical fertilizers which are by-products of the petroleum industry. Fuels are also required for storage and transportation thus directly enhancing the potential transmission effect of oil prices on coffee prices. These results are consistent with the findings of Swaray (2007) and Baffes (2007).

The results from the seasonal dummies indicate that prices during May, June, August, September and October had increasing effect on the current month variability of export price of coffee, while prices during November and December had a decreasing effect. The link between those months and export price volatility is likely to be related to the seasonal pattern of coffee price. Jordaan et al., (2007) stated that fluctuations in prices may be characterized by lower prices at harvest compared to prices during other seasons.

The results also indicate that lagged shocks (represented by ARCH (-1)) of the monthly price of coffee have statistically significant effect on the current month price volatility of coffee. Similarly, EGARCH (-1), EGARCH (-2) and EGARCH (-3) terms are statistically significant at the 1% level. This indicates that the current month price volatility of coffee is affected by its past (lagged) price volatilities.
The asymmetry coefficient in the variance equation is positive and statistically significant. This indicates that positive shocks (unexpected decreases in the export price of coffee) have a more prominent effect on the volatility of the export price of coffee than negative shocks (sharp increases in coffee prices). A study by Maurice & Davis (2011) also revealed that coffee Arabica and Robusta price volatilities are more affected by positive shocks than negative shocks.

3.10 In-sample forecast of price volatility

One of the fundamental applications of developing GARCH family models is forecasting. A plot of the dynamic in-sample forecasts based on the fitted ARMA (1,1)-EGARCH (3,1) model is shown in Figure 3. We can observe that there was high export price volatility of coffee around 2012, 2015 and 2016.

![Forecast of Variance](image)

Figure 3: In-sample forecast of monthly export price volatility of coffee

4. Conclusion and recommendation

4.1 Conclusion

This study has analyzed the export price of coffee and major macroeconomic variables that determine its volatility based on monthly data from January 2002 to December 2016. The results from the fitted EGARCH model revealed that non-food inflation rate, exchange rate and fuel oil price had a significant effect on the volatility of the export price of coffee in Ethiopia. Moreover, recent past shocks and lagged volatilities were found to increase the contemporaneous volatility of the same. Thus, potential stakeholders should work hard to minimize the fluctuation on the aforementioned drivers so that coffee export price volatility could be optimized at a reasonable level.

4.2 Recommendation

Based on the findings of this study, the following recommendations are forwarded:
The volatility of export price returns of coffee was influenced by macroeconomic factors such as exchange rate, non-food inflation rate, and price of crude oil. Therefore, concerned bodies should give due attention to these factors during policy formulation.

Further studies that consider some potential variables that may have significant linkage with price volatility of coffee (such as climatic conditions and world demand-supply related factors) are recommended.

References


